Unit 2 Test Study Guide
(Functions & Their Graphs)

Name: ______________________

Date: ______________________ Per: ________

Topic 1: Evaluating Functions

For questions 1 and 2, evaluate the following, given $f(x)=\dfrac{x-2}{2x+3}$.

1. $f(9)$

$\dfrac{9-2}{2(9)+3}=\dfrac{7}{21}=\boxed{\dfrac{1}{3}}$

2. $f(x-1)$

$\dfrac{x-1-2}{2(x-1)+3}=\boxed{\dfrac{x-3}{2x+1}}$

For questions 3 and 4, evaluate the following, given $g(x)=3x-x^2$.

3. $g(2x-1)$

$3(2x-1)-(2x-1)^2$

$=6x-3-(4x^2-4x+1)$

$=\boxed{-4x^2+10x-4}$

4. $g(-3x)$

$3(-3x)-(-3x)^2$

$=-9x-9x^2$

$=\boxed{-9x^2-9x}$

For questions 5 and 6, evaluate the following, given $h(x)=\begin{cases}|-4x+7| & \text{if } x<-3\\ -x^3+2x^2 & \text{if } x\ge -3\end{cases}$.

Since we are evaluating , h(-7), use this expression because it is the one that goes with x<-3

5. $h(-7)$

$|-4(-7)+7|$

$=|28+7|=\boxed{35}$

6. $h(-3)$

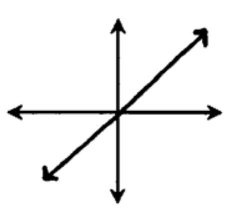
$-(-3)^3+2(-3)^2$

$=27+18=\boxed{45}$

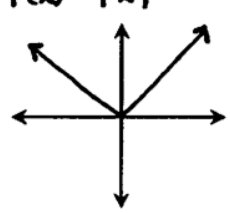
Topic 2: Parent Functions, Transformations, and Graphing

For each function family below, give the parent function and sketch the shape of its graph.

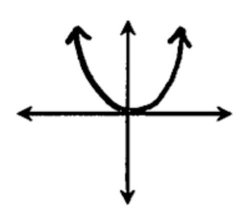
7. Linear $f(x)=x$



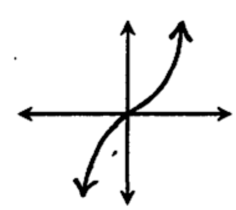
8. Absolute Value $f(x)=|x|$



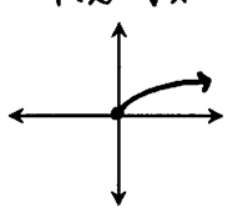
9. Quadratic $f(x)=x^2$



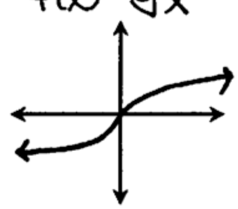
10. Cubic $f(x)=x^3$



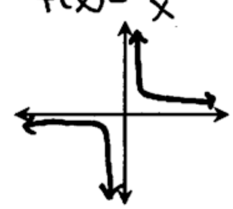
11. Square Root $f(x)=\sqrt{x}$



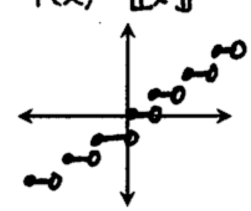
12. Cube Root $f(x)=\sqrt[3]{x}$



13. Reciprocal $f(x)=\frac{1}{x}$



14. Greatest Integer $f(x)=[\![x]\!]$

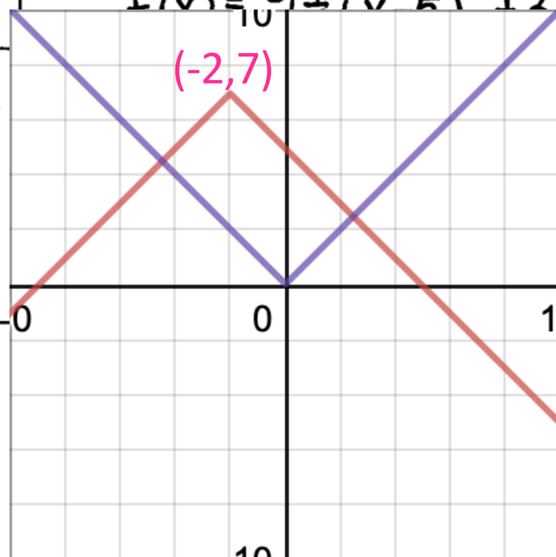


15. If the quadratic parent function is reflected in the y-axis and vertically compressed by a factor of ½, write an equation to represent the **new function.**

$f(x) = \frac{1}{2}(-x)^2$

16. If the cube root parent function is horizontally stretched by a factor of 4, then translated 5 units right and 3 units up, write an equation to represent the **new function.**

$f(x) = \sqrt[3]{\frac{1}{4}(x-5)} + 3$



17. The absolute value parent function has transformations applied such that it creates an absolute maximum at (-2, 7). Write an eqution that could represent this **new function.**

Your "V" shape will be upside down at the max of (-2, 7)

$f(x) = -|x+2| + 7$

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19. Describe all transformations from the parent function given the function below.

$$f(x) = -3\left(\frac{1}{2}x\right)^3 + 7$$

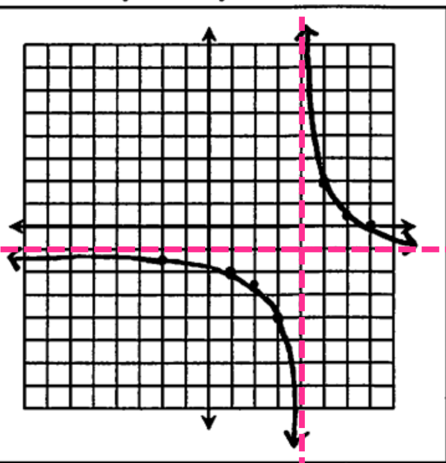
- Vert stretch by 3
- Horiz stretch by 2
- Reflect in x-axis
- Translate up 7

e
below.

- Reflect in y-axis
- Translate right 5, up 2

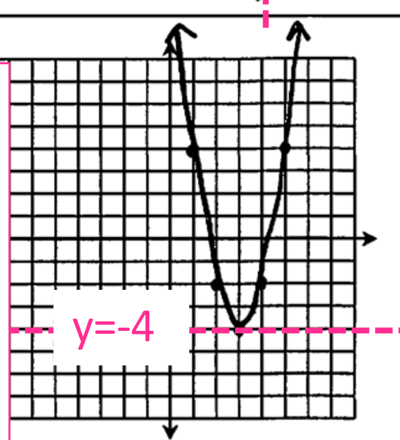
Graph each function and identify all key characteristics.

21. $f(x) = \dfrac{3}{x-4} - 1$



Domain: $\{x \mid x \neq 4\}$	**Range:** $\{y \mid y \neq -1\}$
***x*-int:** (7,0)	***y*-int:** (0, -1.75)
Extrema None	
Increasing Interval: None	
Decreasing Interval: $(-\infty, 4), (4, \infty)$	
End Behavior: As $x \to \infty$, $f(x) \to -1$; As $x \to -\infty$, $f(x) \to -1$	

22. $f(x) = 2(x-3)^2 - 4$

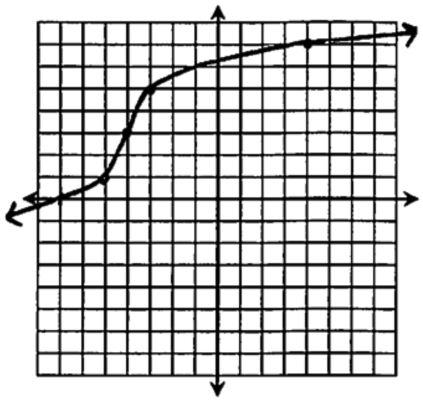


x-int. is where it crosses the x-axis and y =0
Set your equation equal to 0 and solve.

$0=2(x-3)^2 - 4$
$4 = 2(x-3)^2$
$2 = (x-3)^2$
$\sqrt{2} = \sqrt{(x-3)^2}$
$\pm\sqrt{2} = x - 3$
Now solve to get 2 answers for "x"

Domain: $\mathbb{R}$	**Range:** $\{y \mid y \geq -4\}$
***x*-int:** (1.6,0), (4.4,0)	***y*-int:** (0,14)
Extrema (3,-4) - Abs. Minimum	
Increasing Interval: $(3, \infty)$	
Decreasing Interval: $(-\infty, 3)$	
End Behavior: As $x \to \infty$, $f(x) \to \infty$; As $x \to -\infty$, $f(x) \to \infty$	

23. $f(x) = 2\sqrt[3]{(x+4)} + 3$

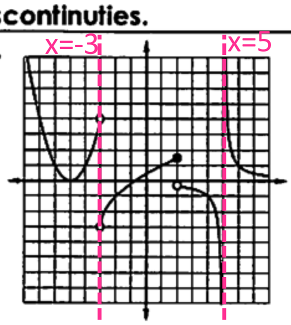


Domain: $\mathbb{R}$	Range: $\mathbb{R}$
x-int: $(-7.375, 0)$	y-int: $(0, 6.17)$
Extrema: None	
Increasing Interval: $(-\infty, \infty)$	
Decreasing Interval: None	
End Behavior: As $x \to \infty$, $f(x) \to \infty$ As $x \to -\infty$, $f(x) \to -\infty$	

Topic 3: Piecewise Functions

Identify the domain and range of each graph below. State the location and type of any discontinuties.

24.

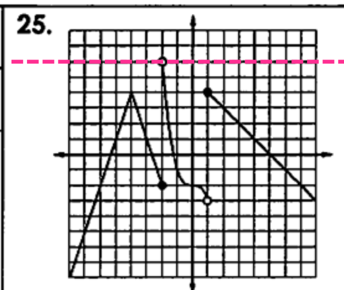


Domain: $\{x \mid x \neq -3, 5\}$

Range: $\mathbb{R}$

Discontinuities:
$x = -3$; jump
$x = 2$; jump
$x = 5$; infinite

25.



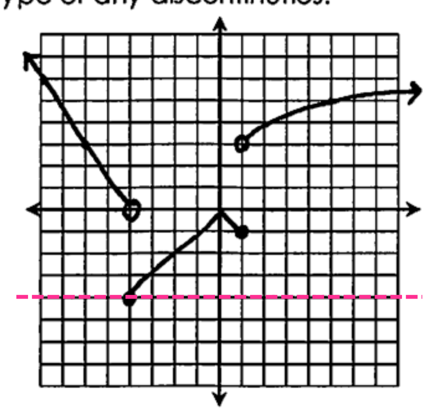
Domain: $\mathbb{R}$

Range: $\{y \mid y < 6\}$

Discontinuities:
$x = -2$; jump
$x = 1$; jump

26. Graph the function below. Identify the domain and range, then, state the location and type of any discontinuties.

$$f(x) = \begin{cases} -\frac{3}{2}x - 6 & \text{if } x < -4 \\ -|x| & \text{if } -4 \le x \le 1 \\ \sqrt{x-1} + 3 & \text{if } x > 1 \end{cases}$$



Domain: $\mathbb{R}$

Range: $\{y \mid y \geq -4\}$

Discontinuities:
$x = -4$; jump
$x = 1$; jump

Topic 4: Average Rate of Change

Find the average rate of change of the function on the given interval.

27. $f(x) = 2x^2 - 3x + 1$; [-3, 2]

$$m = \frac{3-28}{2+3} = \frac{-25}{5} = \boxed{-5}$$

28. $f(x) = \frac{2x-1}{x+3}$; [-10, -5]

$$m = \frac{\frac{11}{2} - 3}{-5 + 10} = \frac{\frac{5}{2}}{5} = \boxed{\frac{1}{2}}$$

29. A football is kicked from a point on the ground such that its height $h(t)$, in feet, is given by the equation $h(t) = -16t^2 + 80t$, where t is time in seconds. Find the average rate of change in the height of the ball from when it reaches its maximum height until it reaches the ground.

$\frac{-b}{2a}$

Height of the ground is 0 so set the equation = 0 and solve.

$0 = -16t^2 + 80t$

$0 = -16t(t-5)$

$-16t = 0$ & $t - 5 = 0$

$t = \frac{-80}{2(-16)} = 2.5$

Max: $(2.5, 100)$

$[2.5, 5]$

$\frac{0-100}{5-2.5} = -\frac{100}{2.5} = \boxed{-40 \text{ ft/sec}}$

Topic 5: Tests for Symmetry / Even & Odd Functions

Use the graph to determine if the relations given below are symmetrical to the x-axis, y-axis, and/or origin. Confirm your answer algebraically.

30. $x^2 - y^2 = 4$

$(-x)^2 - y^2 = 4$

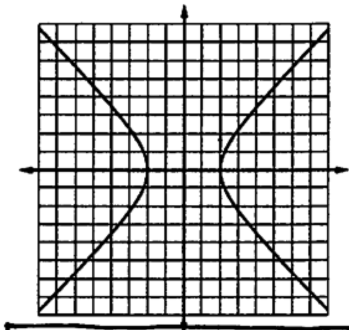
$x^2 - y^2 = 4$ ✓

$x^2 - (-y)^2 = 4$

$x^2 - y^2 = 4$ ✓

$(-x)^2 - (-y)^2 = 4$

$x^2 - y^2 = 4$ ✓



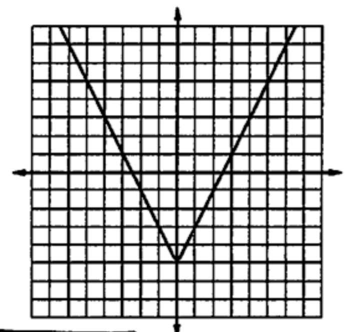
$\boxed{\text{Sym. to x, y, + origin}}$

31. $y = |2x| - 5$

$y = |2(-x)| - 5$

$y = |-2x| - 5$

$y = |2x| - 5$ ✓



$\boxed{\text{sym. to y-axis}}$

Determine whether the function below is even, odd, or neither. Prove your answer algebraically.

32. $f(x) = -3x^3 + 5x$

$f(-x) = -3(-x)^3 + 5(-x)$

$= 3x^3 - 5x$

$\boxed{\text{Odd (sym. to origin)}}$

EVEN

EVEN AND ODD *Functions*	Algebraic Check: $f(-x) = f(x)$
	Algebraic Check: $f(-x) = -f(x)$

ODD

33. $f(x) = 5x^2 + 2x - 1$

$f(-x) = 5(-x)^2 + 2(-x) - 1$

$= 5x^2 - 2x - 1$

$\boxed{\text{Neither}}$

Topic 6: Function Operations & Compositions of Functions

Use $f(x) = 3 - 2x$, $g(x) = \sqrt{x+7}$, and $h(x) = x^2 - 5x$ to find each function below. Be sure to state any domain restrictions, wherever necessary.

34. $(g+f)(x)$

$\sqrt{x+7} + 3 - 2x$

$= \boxed{\sqrt{x+7} - 2x + 3}$

D: $x \geq -7$

35. $(h \cdot f)(x)$

$(x^2 - 5x)(3 - 2x)$

$= \boxed{-2x^3 + 13x^2 - 15x}$

36. $\left(\frac{f}{h}\right)(x)$

$\boxed{\frac{3-2x}{x^2-5x}}$ Can't =0

D: $x \neq 0, 5$

Use $f(x) = -x^2 - 2x$, $g(x) = \sqrt{x+7}$, and $h(x) = 3x - 1$ to find each function below. Give the domain for each.

37. $(h \circ f)(x)$

$3(-x^2-2x)-1$

$= \boxed{-3x^2-6x-1}$

D: $\mathbb{R}$

38. $(f \circ g)(x)$

$-(\sqrt{x+7})^2 - 2(\sqrt{x+7})$

$= -(x+7) - 2\sqrt{x+7}$

$= \boxed{-x-7-2\sqrt{x+7}}$

D: $x \geq -7$

39. $(f \circ h)(x)$

$-(3x-1)^2 - 2(3x-1)$

$= -(9x^2-6x+1) - 6x + 2$

$= \boxed{-9x^2+1}$

D: $\mathbb{R}$

Given $h(x)$ below, find two functions, f and g, such that $(f \circ g)(x) = h(x)$.

40. $h(x) = \frac{5}{x-9} - 2$

The value of g(x) will be put into f(x)

$f(x) = \frac{5}{x} - 2$

Take the expression with the x value in it and make it g(x) then rewrite f(x) using just "x"

$g(x) = x - 9$

41. $h(x) = -\sqrt{2(x+5)} + 7$

$f(x) = -\sqrt{2x} + 7$

$g(x) = x + 5$

Use $f(x) = |10-2x|$, $g(x) = \sqrt[3]{2x-3}$, and $h(x) = \frac{1}{2}x + 5$ to evaluate each function below.

42. $(g - f)(15)$

$g(15) = \sqrt[3]{2(15)-3} = 3$

$f(15) = |10-2(15)| = 20$

$3 - 20 = \boxed{-17}$

43. $\left(\frac{h}{g}\right)(-12)$

$h(-12) = \frac{1}{2}(-12) + 5 = -1$

$g(-12) = \sqrt[3]{2(-12)-3} = -3$

$\frac{-1}{-3} = \boxed{\frac{1}{3}}$

44. $(g \circ h)(-6)$

$h(-6) = \frac{1}{2}(-6) + 5$

$= 2$

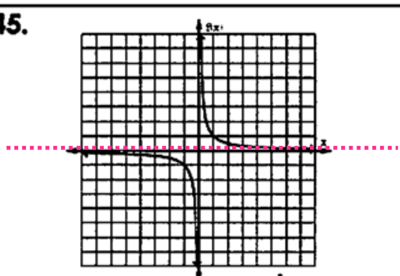
$g(2) = \sqrt[3]{2(2)-3}$

$= \boxed{1}$

Topic 7: Inverse Functions

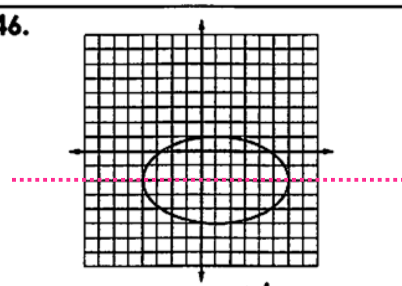
Determine if the graph represents a one-to-one function.

45.



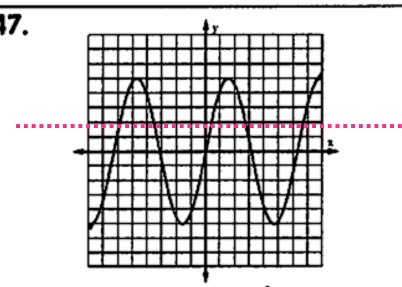
Yes

46.



No

47.



No

Determine if $f(x)$ has an inverse, if yes, find $f^{-1}(x)$. State any restrictions in the domain.

48. $f(x) = \sqrt[3]{x-7} + 2$

$x = \sqrt[3]{y-7} + 2$

$x - 2 = \sqrt[3]{y-7}$

$(x-2)^3 = y - 7$

$\boxed{f^{-1}(x) = (x-2)^3 + 7}$

49. $f(x) = 2|x+5|$

No Inverse;

fails Horiz. Line Test

50. $f(x) = 4x^2 - 7; x \geq 0$

$x = 4y^2 - 7$

$x + 7 = 4y^2$

$\frac{x+7}{4} = y^2$

$\boxed{f^{-1}(x) = \sqrt{\frac{x+7}{4}}}$

$x \geq -7$

51. $f(x) = \frac{x-6}{x+5}$

$x = \frac{y-6}{y+5}$

$xy + 5x = y - 6$

$xy - y = -5x - 6$

$y(x-1) = -5x - 6$

$\boxed{f^{-1}(x) = \frac{-5x-6}{x-1}}$

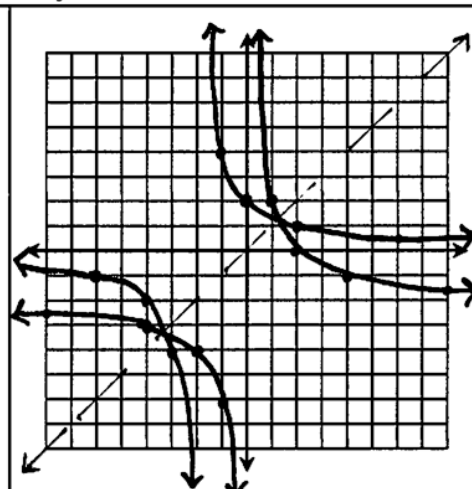
Prove $f(x)$ and $g(x)$ are inverses both algebraically and graphically.

52. $f(x) = \frac{4}{x} - 2$

$g(x) = \frac{4}{x+2}$

$(f \circ g)(x) = \frac{4}{\frac{4}{x+2}} - 2 = x + 2 - 2 = \boxed{x}$

$(g \circ f)(x) = \frac{4}{\frac{4}{x} - 2 + 2} = \frac{4}{\frac{4}{x}} = \boxed{x}$



53. $f(x) = \left(\frac{1}{2}x\right)^3 - 3$

$g(x) = 2\sqrt[3]{x+3}$

$(f \circ g)(x) = \left(\frac{1}{2}\left(2\sqrt[3]{x+3}\right)\right)^3 - 3 = x + 3 - 3 = \boxed{x}$

$(g \circ f)(x) = 2\sqrt[3]{\left(\frac{1}{2}x\right)^3 - 3 + 3} = 2\sqrt[3]{\left(\frac{1}{2}x\right)^3} = \boxed{x}$