

Solving Logarithmic & Exponential Equations Coloring Sheet Answer Key

<p>A. $4^{-4x+5} + 2 = 18$ Pink</p>	<p>Green $\log_5(3x+8) = \log_5(6x-5)$</p>
<p>B. $2 + 81^{x-2} = 11$ Yellow</p>	<p>Purple $\log_4(12x+4) = 3$</p>
<p>C. $25^{x+3} + 10 = 35$ Brown</p>	<p>Yellow $\log_7(16x+9) = 2$</p>
<p>D. $6^{3x+7} = 36^{x+6}$ Purple</p>	<p>Red $\log_3(8x-3) = 3$</p>
<p>E. $4^{x+3} = 16^{2x-5}$ Green</p>	<p>Pink $(4x - 1) \log_9 4 = \log_9 16$</p>
<p>F. $8^{x-6} = \frac{1}{2^{5x-12}}$ Red</p>	<p>Brown $\log_2(8) + \log_2(2-7x) = 7$</p>



LOGARITHMIC & EXPONENTIAL EQUATIONS *Review!*

LOGARITHMIC EQUATIONS

1. $\log_7(x+13) = \log_7(3-x)$

$$x+13 = 3-x$$

$$2x = -10$$

$$x = \{-5\}$$

2. $\log_2(n^2+13) = \log_2(n-1) + \log_2(n+3)$

$$\log_2(n^2+13) = \log_2(n-1)(n+3)$$

$$n^2+13 = n^2+2n-3$$

$$16 = 2n$$

$$n = \{8\}$$

3. $2 \cdot \ln(a+3) = \frac{1}{4} \cdot \ln 16 + \ln(a+7)$

$$\ln(a+3)^2 = \ln 16^{1/4} + \ln(a+7)$$

$$\ln(a^2+6a+9) = \ln(2a+14)$$

$$a^2+6a+9 = 2a+14$$

$$a^2+4a-5 = 0$$

$$(a+5)(a-1) = 0$$

$$a = \{-5, 1\}$$

$$a = \{1\}$$

4. $\log(3c+4) - \log(c-6) = \log(c+6)$

$$\log \frac{3c+4}{c-6} = \log c+6$$

$$\frac{3c+4}{c-6} = c+6$$

$$3c+4 = c^2-36$$

$$0 = c^2-3c-40$$

$$0 = (c-8)(c+5) \quad c = \{-5, 8\}$$

$$c = \{8\}$$

5. $\log_2(5v+23) - 9 = -2$

$$\log_2(5v+23) = 7$$

$$2^7 = 5v+23$$

$$128 = 5v+23$$

$$105 = 5v$$

$$v = \{21\}$$

6. $\log_{16}(p+5) - \log_{16}(p-2) = \frac{1}{2}$

$$\log_{16} \frac{p+5}{p-2} = \frac{1}{2}$$

$$16^{1/2} = \frac{p+5}{p-2}$$

$$4 = \frac{p+5}{p-2}$$

$$4p-8 = p+5$$

$$3p = 13$$

$$p = \{13/3\}$$

7. $\ln(r+1) + 3 \cdot \ln 2 = 7$

$$\ln(r+1) + \ln 2^3 = 7$$

$$\ln_e(8r+8) = 7$$

$$e^7 = 8r+8$$

$$1096.63 = 8r+8$$

$$1088.63 = 8r$$

$$r = \{136.08\}$$

8. $\frac{1}{3} \cdot \log_9 64 + 2 \cdot \log_9 n = 2$

$$\log_9 64^{1/3} + \log_9 n^2 = 2$$

$$\log_9 4n^2 = 2$$

$$4n^2 = 81$$

$$n^2 = 81/4$$

$$n = \{-9/2, 9/2\}$$

$$n = \{9/2\}$$

EXPONENTIAL EQUATIONS

9. $\left(\frac{1}{27}\right)^{2x-6} = 9^{x-1}$

$$3^{-3(2x-6)} = 3^{2(x-1)}$$

$$-3(2x-6) = 2(x-1)$$

$$-6x + 18 = 2x - 2$$

$$20 = 8x$$

$$\boxed{\frac{5}{2} = x}$$

10. $4^{3m+1} = \left(\frac{1}{8}\right)^{m+4} \cdot 32^{m-2}$

$$2^{2(3m+1)} = 2^{-3(m+4)} \cdot 2^{5(m-2)}$$

$$6m+2 = -3m-12+5m-10$$

$$6m+2 = 2m-22$$

$$4m = -24$$

$$\boxed{m = -6}$$

11. $5^{w-1} = 90$

$$\log 5^{w-1} = \log 90$$

$$\frac{(w-1) \cdot \log 5}{\log 5} = \frac{\log 90}{\log 5}$$

$$w-1 = 2.7959$$

$$\boxed{w = 3.7959}$$

12. $e^{3r-2} - 16 = 120$

$$e^{3r-2} = 136$$

$$\ln e^{3r-2} = \ln 136$$

$$3r-2 = 4.9127$$

$$3r = 6.9127$$

$$\boxed{r = 2.3042}$$

13. $-4 \cdot 9^{2k+5} + 14 = 6$

$$-4 \cdot 9^{2k+5} = -8$$

$$9^{2k+5} = 2$$

$$\log 9^{2k+5} = \log 2$$

$$\frac{(2k+5) \cdot \log 9}{\log 9} = \frac{\log 2}{\log 9}$$

$$2k+5 = .3155$$

$$2k = -4.6845$$

$$\boxed{k = -2.3423}$$

14. $\frac{2}{3} \cdot 5^{m-8} - 9 = 21$

$$\frac{2}{3} \cdot 5^{m-8} = 30$$

$$5^{m-8} = 45$$

$$\log 5^{m-8} = \log 45$$

$$\frac{(m-8) \cdot \log 5}{\log 5} = \frac{\log 45}{\log 5}$$

$$m-8 = 2.3652$$

$$\boxed{m = 10.3652}$$

15. $3^{4x+1} = 8^{x-5}$

$$\log 3^{4x+1} = \log 8^{x-5}$$

$$(4x+1) \cdot \log 3 = (x-5) \cdot \log 8$$

$$4x \cdot \log 3 + \log 3 = x \cdot \log 8 - 5 \cdot \log 8$$

$$4x \cdot \log 3 - x \cdot \log 8 = -5 \cdot \log 8 - \log 3$$

$$\frac{x(4 \cdot \log 3 - \log 8)}{4 \cdot \log 3 - \log 8} = \frac{-5 \cdot \log 8 - \log 3}{4 \cdot \log 3 - \log 8}$$

$$\boxed{x = -4.9658}$$

16. $4^{2x+3} = 7^{15-2x}$

$$\log 4^{2x+3} = \log 7^{15-2x}$$

$$(2x+3) \cdot \log 4 = (15-2x) \cdot \log 7$$

$$2x \cdot \log 4 + 3 \cdot \log 4 = 15 \cdot \log 7 - 2x \cdot \log 7$$

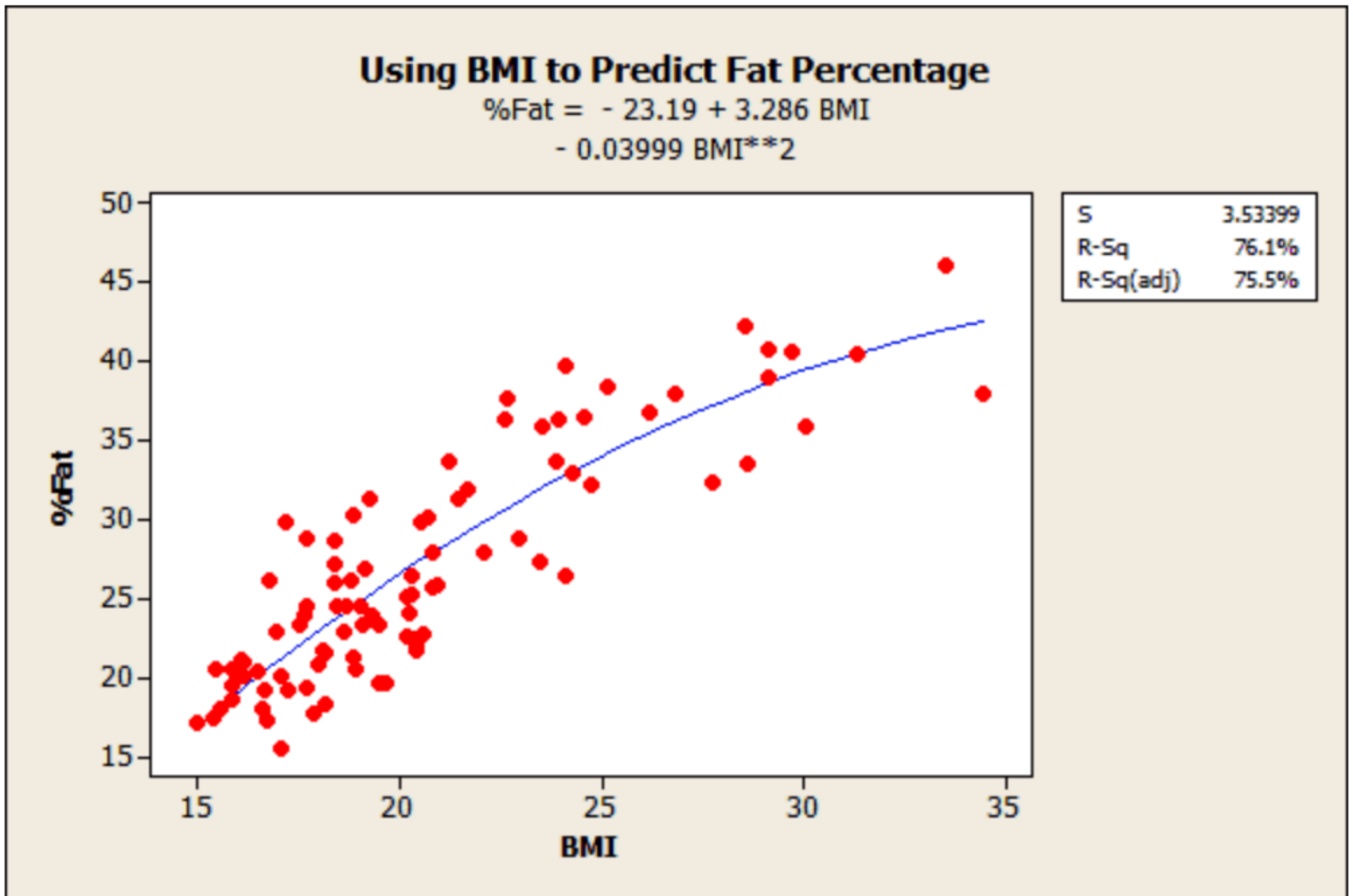
$$2x \cdot \log 4 + 2x \cdot \log 7 = 15 \cdot \log 7 - 3 \cdot \log 4$$

$$\frac{2x(\log 4 + \log 7)}{\log 4 + \log 7} = \frac{15 \cdot \log 7 - 3 \cdot \log 4}{\log 4 + \log 7}$$

$$2x = 7.5115$$

$$\boxed{x = 3.7557}$$

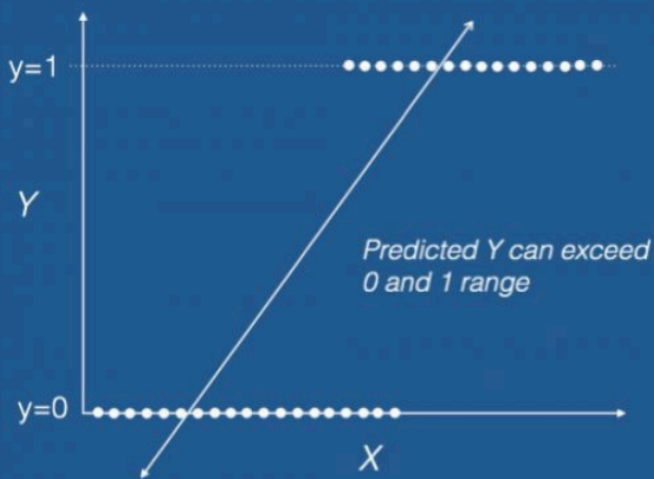
A *scatter plot* is a set of points on a grid, used to visualize a possible trend in the data.



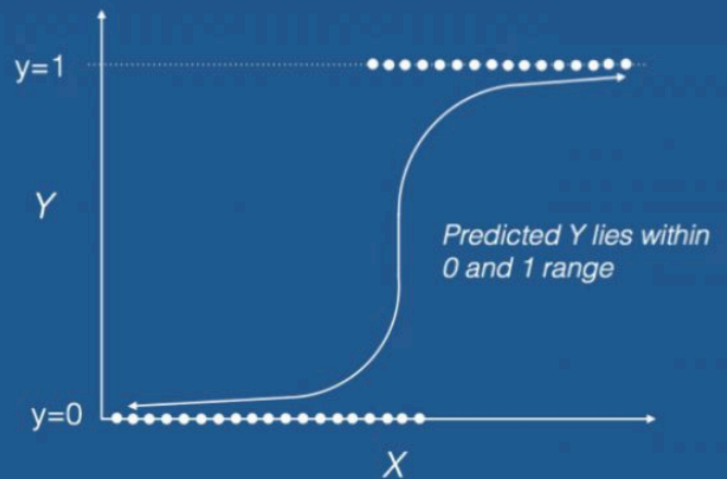
Power Regression is one in which the response variable is proportional to the explanatory variable raised to a **power**.

Linear regression works when the outcome is continuous (if this, than this). Logistic regression works when the outcome is binary. Trying to use linear regression on a binary outcome variable simply won't work (well).

Linear Regression



Logistic Regression



Logistic regression generates a model that allows you to predict the *probability of success*, given a certain x-value. Data that you put into the model will only include actual outcomes (at a given X value, a success either **was** observed or it **was not**).

Kinds of problems you can use logistic regression for.

Spam Detection : Predicting if an email is Spam or not

Credit Card Fraud : Predicting if a given credit card transaction is fraud or not

Health : Predicting if a given mass of tissue is benign or malignant

Marketing : Predicting if a given user will buy an insurance product or not

Banking : Predicting if a customer will default on a loan.

Show me!

Caffeine is found in coffee, tea, and soft drinks. Many people find that caffeine makes it difficult for them to sleep. The following data was collected in a study to determine how quickly the human body metabolizes caffeine. Each person started with 200 mg of caffeine in her or his bloodstream, and the caffeine level was measured at various times.

Determine the time it takes for an average person to metabolize 50% of the caffeine in her or his bloodstream. Round your answer to the nearest tenth of an hour.

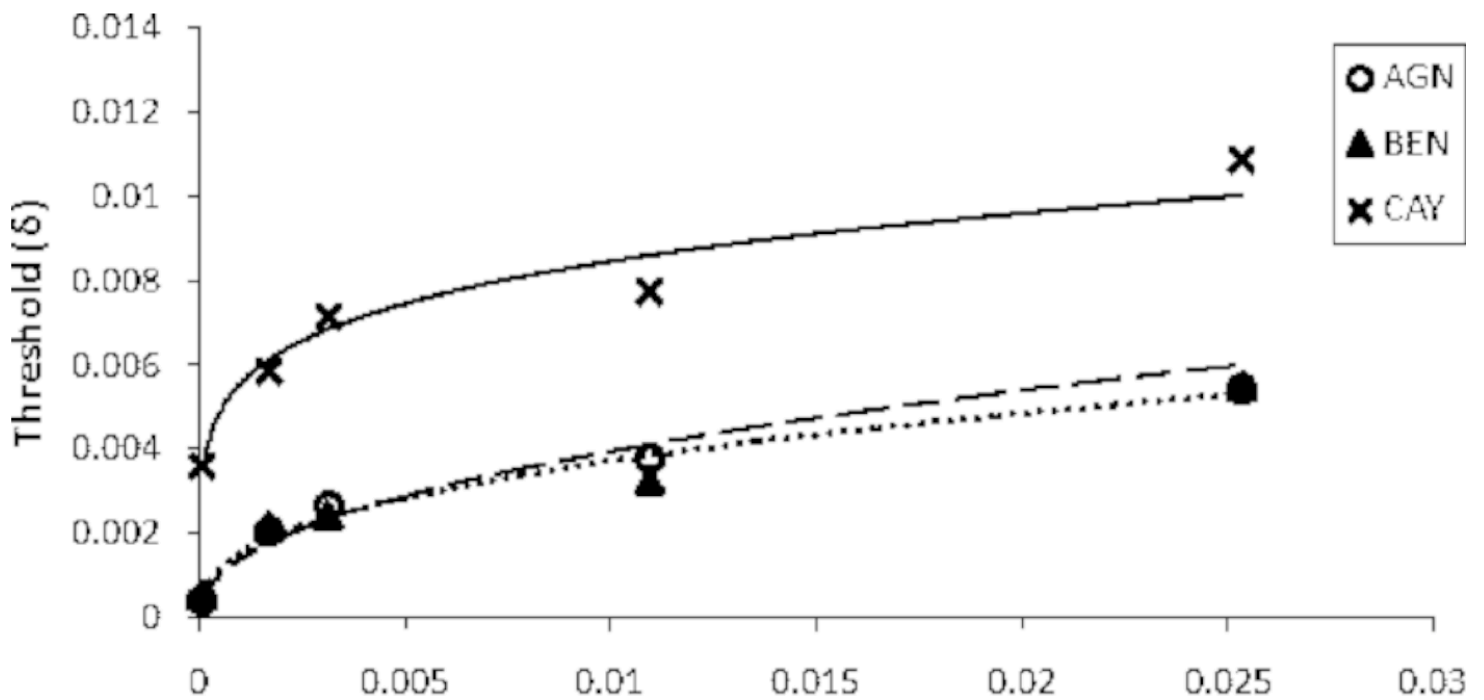
Paula drank a cup of coffee what contained 200mg of caffeine at 10:00 am. How much caffeine will be in her bloodstream 11 hours later, at 9:00 pm that evening? Round your answer to the nearest milligram.

Caffeine Level in Bloodstream, c (in mg)	Time after Ingesting, t (in hours)
167	1.5
113	5
33	14
80	7.5
145	3
49	12.5
21	19
112	5
25	17.5
76	9

A logarithmic regression function is the equation of a curve of best fit when the scatter plot looks like a logarithmic function.

Using a calculator, the regression uses natural logs,

$$y = a + b \ln x$$



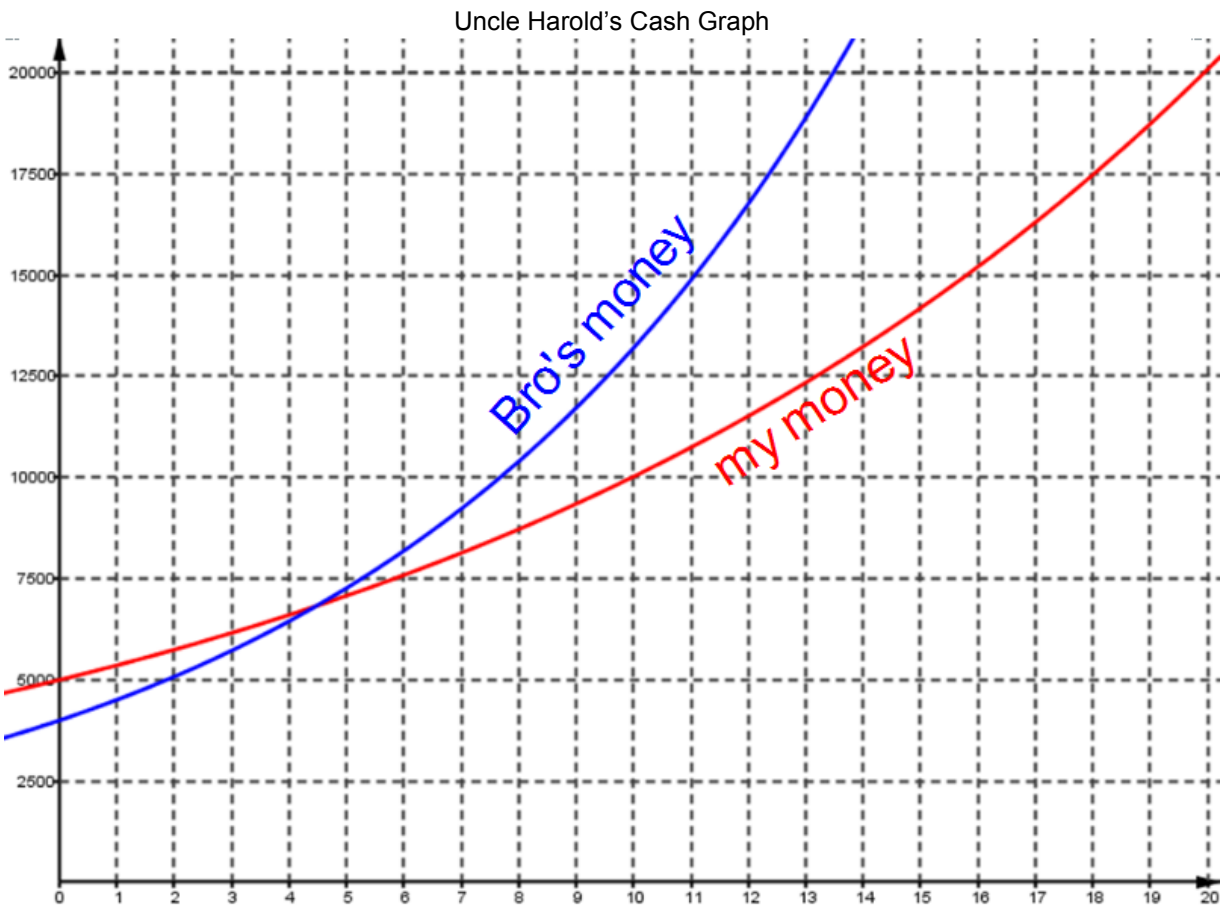
MONETARY GROWTH

You inherit \$5,000 from your long lost Uncle Harold. The bad news is that the money must sit in a bank account for the next ten years until you can use it. The account earns 7.2% interest, compounded annually. This means that you will need to multiply the amount of money by 1.072 to determine how much money remains at the end of the next year.

Fill in the chart below to determine how much money is in the account at the end of each year.
Round to the nearest penny!

# of years	0	1	2	3	4	5	6
\$ in account	5000	5360	5745.92	6159.63	6603.12	7078.54	7588.20

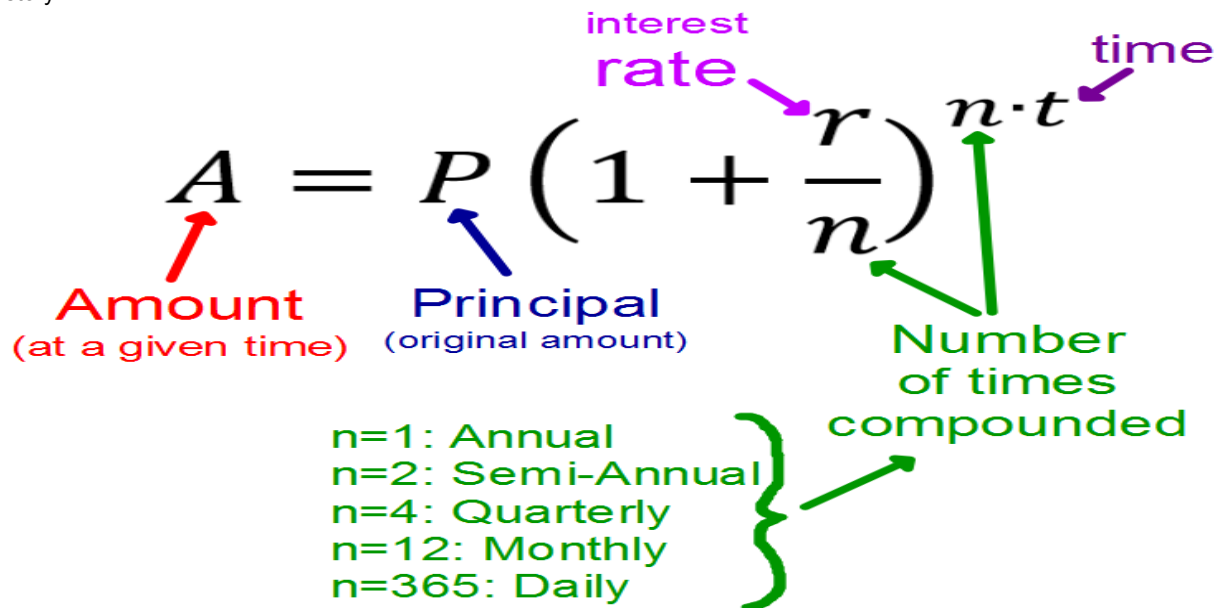
Use the chart above to create a graph.



According to the chart, approximately when will your original inheritance double in size?

10 years

The compounded interest formula is pictured below. Create a formula using the information from the Uncle Harold story.



Uncle Harold Formula:

Use this formula to find the balance (the amount of money that would be in the account) at the end of ten years.

equation: $A = 5000 \left(1 + \frac{.072}{1} \right)^{10}$ balance: \$10,021.16

Use this formula to find the balance if you let all of the money earn interest for 20 years. (Show work below.)

equation: $A = 5000 \left(1 + \frac{.072}{1} \right)^{20}$ balance: \$20,084.72

Plot these two points on the Uncle Harold's Cash Graph and connect all of the points on the graph. Label this line: "My Money"

You are surprised to learn that your brother also received money from Uncle Harold. He only received \$4000, but his account is earning 12% interest, compounded monthly.

Write the equation that represents the money in your brother's account over time.

$A = 4000 \left(1 + \frac{.12}{12} \right)^{(12t)}$ Fill in the chart below, using the equation above. Remember that you should type (n·t) in parenthesis!

# of years	0	2	4	6	8	10	12
\$ in account	4000	5078.94	6448.90	8188.40	10,397.09	13,201.55	16,762.46

Plot the points on the Uncle Harold's Cash graph. Connect the points with a line, and label it: "BRO'S MONEY".

Approximately when will your brother's account have more money than yours?

Between 4 and 5 years.