Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
COMPLEX ZEROS	Just as polynomial function can have real zeros (rational and/or irrational), it can also have **complex zeros.** Find all zeros of the polynomial functions below. Simplify all irraitonal and complex solutions.

1. $f(x)=x^4-4x^2-12$

$f(x)=(x^2-6)(x^2+2)$

$x^2=6$ | $x^2=-2$

$x=\pm\sqrt{6}$ | $x=\pm i\sqrt{2}$

$\boxed{x=\{\pm i\sqrt{2}, \pm\sqrt{6}\}}$

2. $f(x)=4x^4+31x^2-8$

$f(x)=(4x^2-1)(x^2+8)$

$4x^2=1$ | $x^2=-8$

$x^2=1/4$ | $x=\pm\sqrt{-8}$

$x=\pm 1/2$ | $x=\pm 2i\sqrt{2}$

$\boxed{x=\{\pm 2i\sqrt{2}, \pm\frac{1}{2}\}}$

3. $f(x)=x^4-16$

$f(x)=(x^2+4)(x^2-4)$

$x^2=-4$ | $x^2=4$

$x=\pm\sqrt{-4}$ | $x=\pm 2$

$x=\pm 2i$

$\boxed{x=\{\pm 2i, \pm 2\}}$

4. $f(x)=-2x^3-16$

$f(x)=-2(x^3+8)$

$f(x)=-2(x+2)(x^2-2x+4)$

$x=-2$ | $x=\frac{2\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$

$x=\frac{2\pm\sqrt{-12}}{2}$

$x=\frac{2\pm 2i\sqrt{3}}{2}$

$\boxed{x=\{-2, 1\pm i\sqrt{3}\}}$

5. $f(x)=x^3-2x^2+x-2$

$f(x)=x^2(x-2)+1(x-2)$

$f(x)=(x^2+1)(x-2)$

$x^2=-1$ | $x=2$

$x=\pm i$

$\boxed{x=\{\pm i, 2\}}$

6. $f(x)=3x^3-4x^2+36x-48$

$f(x)=x^2(3x-4)+12(3x-4)$

$f(x)=(x^2+12)(3x-4)$

$x^2=-12$ | $3x=4$

$x=\pm\sqrt{-12}$ | $x=4/3$

$x=\pm 2i\sqrt{3}$

$\boxed{x=\{\pm 2i\sqrt{3}, \frac{4}{3}\}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
COMPLEX ZEROS	Just as a polynomial function can have real zeros (rational and/or irrational), it can also have **complex zeros**. Find all zeros of the polynomial functions below. Simplify all irrational and complex solutions.	
	1. $f(x) = x^4 - 4x^2 - 12$	**2.** $f(x) = 4x^4 + 31x^2 - 8$
	3. $f(x) = x^4 - 16$	**4.** $f(x) = -2x^3 - 16$
	5. $f(x) = x^3 - 2x^2 + x - 2$	**6.** $f(x) = 3x^3 - 4x^2 + 36x - 48$

	7. $f(x) = 2x^5 - 5x^4 - 2x + 5$ $f(x) = x^4(2x-5) - 1(2x-5)$ $f(x) = (x^4-1)(2x-5)$ $f(x) = (x^2+1)(x^2-1)(2x-5)$ $x^2=-1$, $x = \pm\sqrt{-1}$, $x = \pm i$ | $x^2 = 1$, $x = \pm 1$ | $2x = 5$, $x = \frac{5}{2}$ $\boxed{x = \{\pm i, \pm 1, \frac{5}{2}\}}$	**8.** $f(x) = x^4 + 7x^3 - x - 7$ $f(x) = x^3(x+7) - 1(x+7)$ $f(x) = (x^3-1)(x+7)$ $f(x) = (x-1)(x^2+x+1)(x+7)$ $x=1$ | $x = \frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2(1)}$, $x = \frac{-1 \pm \sqrt{-3}}{2}$, $x = \frac{-1 \pm i\sqrt{3}}{2}$ | $x=-7$ $\boxed{x = \{-7, 1, \frac{-1 \pm i\sqrt{3}}{2}\}}$
Using the Rational Zero Theorem	**Directions:** List all possible rational zeros. Then, find all zeros. **9.** $f(x) = 2x^3 - 5x^2 + 8x - 20$ Possible: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{2}, \pm \frac{5}{2}$ $\frac{5}{2}$ | 2 -5 8 -20 / ↓ 5 0 20 / 2 0 8 0 $f(x) = (2x-5)(2x^2+8)$ $f(x) = 2(2x-5)(x^2+4)$ $2x = 5$, $x = \frac{5}{2}$ | $x^2 = -4$, $x = \pm 2i$ $\boxed{x = \{\frac{5}{2}, \pm 2i\}}$	
	10. $f(x) = x^3 - 2x^2 + 16x + 48$ Possible: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$ -2 | 1 -2 16 48 / ↓ -2 8 -48 / 1 -4 24 0 $f(x) = (x+2)(x^2-4x+24)$ $x=2$ | $x = \frac{4 \pm \sqrt{(-4)^2-4(1)(24)}}{2(1)}$, $x = \frac{4 \pm \sqrt{-80}}{2}$, $x = \frac{4 \pm 4i\sqrt{5}}{2}$ $\boxed{x = \{2, 2 \pm 2i\sqrt{5}\}}$	

4. $f(x) = x^4 - 81$

$f(x) = (x^2+9)(x^2-9)$

(x^2+9)	(x^2-9)
$x^2 = -9$	$x^2 = 9$
$x = \pm 3i$	$x = \pm 3$

$\boxed{x = \{\pm 3i, \pm 3\}}$ $\boxed{f(x) = (x+3i)(x-3i)(x+3)(x-3)}$

5. $f(x) = x^3 - 5x^2 + 16x - 80$

$f(x) = x^2(x-5) + 16(x-5)$

$f(x) = (x^2+16)(x-5)$

(x^2+16)	$(x-5)$
$x^2 = -16$	$x = 5$
$x = \pm 4i$	

$\boxed{x = \{\pm 4i, 5\}}$ $\boxed{f(x) = (x+4i)(x-4i)(x-5)}$

6. $f(x) = x^3 + 6x^2 - 14x + 16$

Possible: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrr} -8 & 1 & 6 & -14 & 16 \\ & \downarrow & -8 & 16 & -16 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$f(x) = (x+8)(x^2-2x+2)$

$(x+8)$	(x^2-2x+2)
$x = -8$	$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$
	$x = \frac{2 \pm \sqrt{-4}}{2}$

$x = \frac{2 \pm 2i}{2}$

$\boxed{x = \{1 \pm i, -8\}}$

$\boxed{f(x) = (x-(1+i))(x-(1-i))(x+8)}$

7. $f(x) = x^3 - 11x + 20$

Possible: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -11 & 20 \\ & \downarrow & -4 & 16 & -20 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$f(x) = (x+4)(x^2-4x+5)$

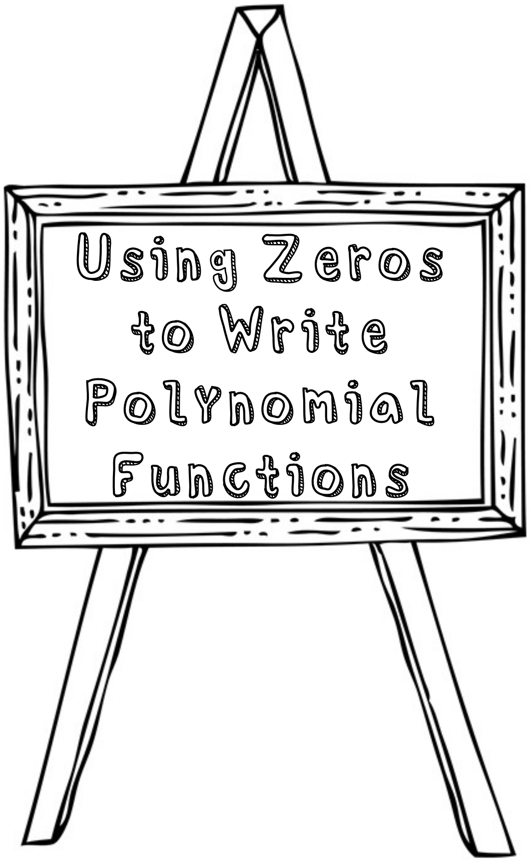
$(x+4)$	(x^2-4x+5)
$x = -4$	$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$
	$x = \frac{4 \pm \sqrt{-4}}{2}$

$x = \frac{4 \pm 2i}{2}$

$\boxed{x = \{2 \pm i, -4\}}$

$\boxed{f(x) = (x-(2+i))(x-(2-i))(x+4)}$

Main Ideas/Questions	Notes/Examples	
	Directions: Write an equation that could represent the function with the given zeros.	
USING ZEROS to Write Polynomial Functions	**1.** 1, 2, 5 $(x-1)(x-2)(x-5)$ $(x-1)(x^2-7x+10)$ $x^3-7x^2+10x-x^2+7x-10$ $\boxed{f(x)=x^3-8x^2+17x-10}$	**2.** –7, –1, 3 $(x+7)(x+1)(x-3)$ $(x+7)(x^2-2x-3)$ $x^3-2x^2-3x+7x^2-14x-21$ $\boxed{f(x)=x^3+5x^2-17x-21}$
	3. $-2, -\frac{4}{3}, 2$ $(x+2)(3x+4)(x-2)$ $(3x+4)(x^2-4)$ $\boxed{f(x)=3x^3+4x^2-12x-16}$	**4.** $\pm\sqrt{2}, 1$ $(x+\sqrt{2})(x-\sqrt{2})(x-1)$ $(x^2-2)(x-1)$ $\boxed{f(x)=x^3-x^2-2x+2}$
	5. $\pm 2\sqrt{3}, \pm 1$ $(x+2\sqrt{3})(x-2\sqrt{3})(x+1)(x-1)$ $(x^2-12)(x^2-1)$ $\boxed{f(x)=x^4-13x^2+12}$	**6.** $-6, 2\pm\sqrt{5}$ $(x+6)(x-(2+\sqrt{5}))(x-(2-\sqrt{5}))$ $(x+6)[x^2-x(2-\sqrt{5})-x(2+\sqrt{5})+(2+\sqrt{5})(2-\sqrt{5})]$ $(x+6)[x^2-2x+x\sqrt{5}-2x-x\sqrt{5}+4-5]$ $(x+6)(x^2-4x-1)$ $\boxed{f(x)=x^3+2x^2-25x-6}$
Examples with **MULTIPLICITY**	**7.** 3 (mult. 2), 5 $(x-3)(x-3)(x-5)$ $(x-3)(x^2-8x+15)$ $x^3-8x^2+15x-3x^2+24x-45$ $\boxed{f(x)=x^3-11x^2+39x-45}$	**8.** $0, 1, \frac{5}{2}$ (mult. 2) $x(x-1)(2x-5)(2x-5)$ $(x^2-x)(4x^2-20x+25)$ $4x^4-20x^3+25x^2-4x^3+20x^2-25x$ $\boxed{f(x)=4x^4-24x^3+45x^2-25x}$



- Set each zero into (…) with opposite signs.

- Factor the expressions

Example:
zeros are 1, 2, 3
$(x-1)(x-2)(x-3)$

$(x-1)(x^2-5x+6)$
$x^3 - 5x^2 + 6x - x^2 + 5x - 6$

$f(x) = x^3 - 6x^2 + 11x - 6$

Write an equation that could represent a function with the following zeros.

1. $1, 2, 5$	**2.** $-7, -1, 3$
3. $-2, -\frac{4}{3}, 2$	**4.** $\pm\sqrt{2}, 1$

Name: ______________________ **Pre-Calculus**

Date: ______________ Per: ______ **Unit 3:** Power, Polynomial, and Rational Functions

Quiz 3-3: Rational, Irrational, and Complex Zeros

Use the Rational Zero Theorem to list all possible rational zeros.

1. $f(x) = x^3 + 11x^2 - 15x - 27$

Possible Rational Zeros:
$\pm 1, \pm 3, \pm 9, \pm 27$

2. $f(x) = 3x^4 - x^3 - 63x^2 - 39x + 20$

Possible Rational Zeros:
$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{3}, \pm 2/3, \pm 4/3, \pm 5/3, \pm 10/3, \pm 20/3$

Give the possible number of positive and negative real zeros using Descartes' Rule.

3. $f(x) = 2x^4 - x^3 - 2x^2 + x$

$f(-x) = 2x^4 + x^3 - 2x^2 - x$

Positive Real Zeros: 2 or 0

Negative Real Zeros: 1

4. $f(x) = 9x^5 - 3x^4 + 10x^3 - x^2 + 27x - 9$

$f(-x) = -9x^5 - 3x^4 - 10x^3 - x^2 - 27x - 9$

Positive Real Zeros: 5, 3, or 1

Negative Real Zeros: 0

Find all zeros. Use the Rational Zero Theorem and synthetic substitution when necessary. Then, give the complete factorization of the function.

5. $f(x) = x^3 - 19x + 30$

Possible: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -19 & 30 \\ & \downarrow & -5 & 25 & -30 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$f(x) = (x+5)(x^2 - 5x + 6)$

$f(x) = (x+5)(x-3)(x-2)$

Zeros:
$x = \{-5, 2, 3\}$

Complete Factorization
$f(x) = (x+5)(x-3)(x-2)$

6. $f(x) = 9x^3 + 63x^2 - 16x - 112$

$f(x) = 9x^2(x+7) - 16(x+7)$

$f(x) = (9x^2 - 16)(x+7)$

$9x^2 = 16$ | $x = -7$

$x^2 = \frac{16}{9}$

$x = \pm \frac{4}{3}$

Zeros:
$x = \{-7, \pm \frac{4}{3}\}$

Complete Factorization
$f(x) = (3x+4)(3x-4)(x+7)$

7. $f(x) = 2x^4 - 9x^3 - 20x^2 + 12x$

$f(x) = x(2x^3 - 9x^2 - 20x + 12)$

$f(x) = x(x+2)(2x^2 - 13x + 6)$

$f(x) = x(x+2)(2x-1)(x-6)$

Possible: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} -2 & 2 & -9 & -20 & 12 \\ & \downarrow & -4 & 26 & -12 \\ \hline & 2 & -13 & 6 & 0 \end{array}$$

Zeros:
$x = \{-2, 0, \frac{1}{2}, 6\}$

Complete Factorization
$f(x) = x(x+2)(2x-1)(x-6)$

8. $f(x) = x^4 + 2x^3 - 2x^2 - 6x - 3$

$f(x) = (x+1)(x^3 + x^2 - 3x - 3)$

$f(x) = (x+1)(x^2-3)(x+1)$

$x=-1$	$x^2=3$	$x=-1$
	$x=\pm\sqrt{3}$	

Possible: $\pm 1, \pm 3$

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -2 & -6 & -3 \\ & \downarrow & -1 & -1 & 3 & 3 \\ \hline & 1 & 1 & -3 & -3 & 0 \end{array}$$

Zeros:
$x = \{-1, \pm\sqrt{3}\}$

Complete Factorization
$f(x) = (x+1)(x+1)(x+\sqrt{3})(x-\sqrt{3})$

9. $f(x) = 5x^3 + 2x^2 - 90x - 36$

$f(x) = x^2(5x+2) - 18(5x+2)$

$f(x) = (x^2-18)(5x+2)$

$x^2=18$	$5x=-2$
$x^2=\sqrt{18}$	$x=-2/5$
$x=\pm 3\sqrt{2}$	

Zeros:
$x = \{-\frac{2}{5}, \pm 3\sqrt{2}\}$

Complete Factorization
$f(x) = (5x+2)(x+3\sqrt{2})(x-3\sqrt{2})$

10. $f(x) = x^4 - x^2 - 20$

$f(x) = (x^2-5)(x^2+4)$

$x^2=5$	$x^2=-4$
$x=\pm\sqrt{5}$	$x=\pm 2i$

Zeros:
$x = \{\pm\sqrt{5}, \pm 2i\}$

Complete Factorization
$f(x) = (x+\sqrt{5})(x-\sqrt{5})(x+2i)(x-2i)$

11. $f(x) = x^3 - 5x^2 - 7x + 51$

Possible: $\pm 1, \pm 3, \pm 17, \pm 51$

$$\begin{array}{r|rrrr} -3 & 1 & -5 & -7 & 51 \\ & \downarrow & -3 & 24 & -51 \\ \hline & 1 & -8 & 17 & 0 \end{array}$$

$f(x) = (x+3)(x^2 - 8x + 17)$

$x = -3$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(17)}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{-4}}{2}$

$x = \dfrac{8 \pm 2i}{2}$

Zeros:
$x = \{-3, 4 \pm i)$

Complete Factorization
$f(x) = (x+3)(x-(4+i))(x-(4-i))$

Write a polynomial function in standard form given the zeros. Write your answers in the box below.

12. -3 (mult. 2), 4 (mult. 2)

$(x+3)(x+3)(x-4)(x-4)$

$(x^2 + 6x + 9)(x^2 - 8x + 16)$

$x^4 - 8x^3 + 16x^2 + 6x^3 - 48x^2 + 96x + 9x^2 - 72x + 144$

$f(x) = x^4 - 2x^3 - 23x^2 + 24x + 144$

13. $\pm\sqrt{6}, \pm\frac{2}{3}, 0$

$(x+\sqrt{6})(x-\sqrt{6})(3x+2)(3x-2)(x)$

$(x^2 - 6)(9x^2 - 4)(x)$

$(9x^4 - 58x^2 + 24)(x)$

$f(x) = 9x^5 - 58x^3 + 24x$

14. $\pm 4\sqrt{2}, 3i$

$(x + 4\sqrt{2})(x - 4\sqrt{2})(x + 3i)(x - 3i)$

$(x^2 - 32)(x^2 + 9)$

$f(x) = x^4 - 23x^2 - 288$

15. $-\frac{1}{2}, -5 + i$

$(2x+1)(x - (-5+i))(x - (-5-i))$

$(2x+1)(x^2 - x(-5-i) - x(-5+i) + (-5+i)(-5-i))$

$(2x+1)(x^2 + 5x + xi + 5x - xi + 26)$

$(2x+1)(x^2 + 10x + 26)$

$2x^3 + 20x^2 + 52x + x^2 + 10x + 26$

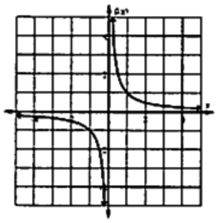
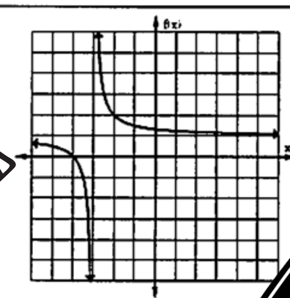
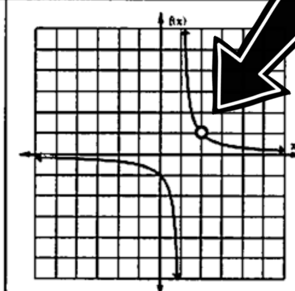
$f(x) = 2x^3 + 21x^2 + 62x + 26$

12. $f(x) = x^4 - 2x^3 - 23x^2 + 24x + 144$

13. $f(x) = 9x^5 - 58x^3 + 24x$

14. $f(x) = x^4 - 23x^2 - 288$

15. $f(x) = 2x^3 + 21x^2 + 62x + 26$

Main Ideas/Questions	Notes/Examples	
RATIONAL Functions 	$$f(x) = \frac{p(x)}{q(x)}$$ • where p(x) and q(x) are polynomial functions and **q(x) ≠ 0** • Simplest form is f(x) = $\frac{1}{x}$	
Vertical & Horizontal **ASYMPTOTES**	• *A line which the graph approaches.* • **Example:** vertical asymptote is x=-3 and • the horizontal is y=1	
HOLES	• A **hole** is a removable discontinuity, or break, in the graph. • If $p(x)$ and $q(x)$ have a **common factor** of $(x - a)$, then there will be a hole in the graph at $x = a$. • **Example:** The function $f(x) = \frac{(x-2)}{(x-2)(x-1)}$ has a hole at X =2.	

Graphing a RATIONAL FUNCTION

❶	Rewrite the function in **FACTORED FORM,** and simplify.
❷	Find the ***x*-intercept(s)** by finding the zeros of $p(x)$.
❸	Find the ***y*-intercept** by finding $f(0)$.
❹	Find the **vertical asymptote(s)** by finding the zeros of $q(x)$.
❺	Find the **horizontal asymptote** using the rules below:
❻	Identify any **hole(s)** in the function by looking at common factors.

degree of p > degree of q	No horizontal asymptote
degree of p < degree of q	x-axis; y=0
degree of p = degree of q	$y = \frac{\text{leading coeff. of } p(x)}{\text{leading coeff. of } q(x)}$

RATIONAL *Functions*

EQUATION FORM: $f(x) = \frac{p(x)}{q(x)}$

Write the function in factored form, simplify, then find the:

X-INTERCEPTS:

Y-INTERCEPT:

VERTICAL ASYMPTOTES:

HORIZONTAL ASYMPTOTE:

IF DEGREE OF P

> THAN DEGREE OF Q: NO HORIZ. ASYM.

< THAN DEGREE OF Q: X-AXIS, Y=0

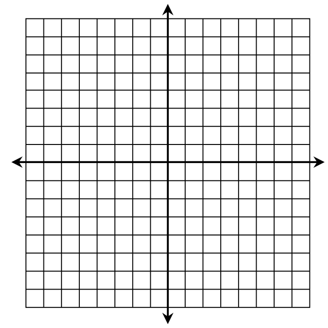
= DEGREE OF Q: $y = \frac{\text{lead coeff of } p(x)}{\text{lead coeff of } q(x)}$

SLANT ASYMPTOTES:

HOLES:

EXAMPLES

❶ $f(x) = \frac{3x+2}{x+2}$



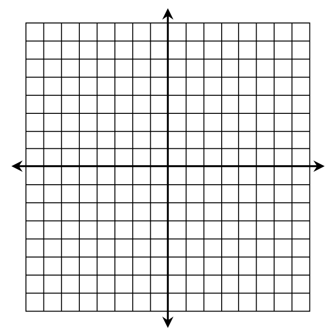
D: ______________________ R: ______________________

x-int(s): ______________________ y-int: __________

VA: ______________________ HA: __________

SA: ______________________ Hole(s): ______________________

❷ $f(x) = \frac{6x-18}{x^2-4x+3}$



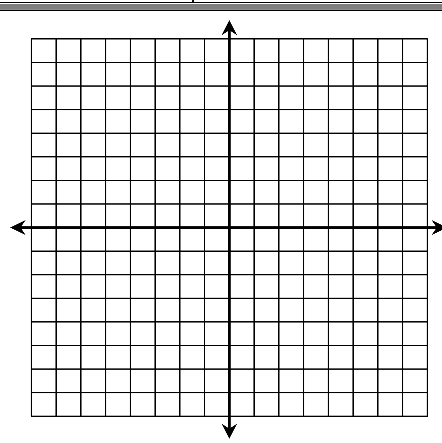
D: ______________________ R: ______________________

x-int(s): ______________________ y-int: __________

VA: ______________________ HA: __________

SA: ______________________ Hole(s): ______________________

. $f(x) = \frac{3x+3}{x+2}$



Domain:

Range:

***x*-int(s):**

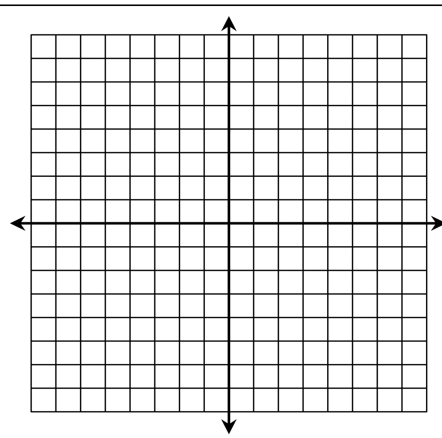
***y*-int:**

VA:

HA:

Hole(s):

. $f(x) = \frac{x^2-2x-15}{x+3}$



Domain:

Range:

***x*-int(s):**

***y*-int:**

VA:

HA:

Hole(s):

RATIONAL *Functions*

EQUATION FORM: $f(x) = \frac{p(x)}{q(x)}$

Write the function in factored form, simplify, then find the:

X-INTERCEPTS: Find the zeros of p(x)

Y-INTERCEPT: Find f(0)

VERTICAL ASYMPTOTES:
Find the zeros of q(x)

HORIZONTAL ASYMPTOTE:

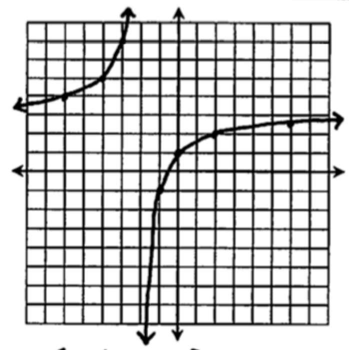
- If degree of p > degree of q: No horiz. asymptote
- If degree of p < degree of q: x-axis; $y=0$
- If degree of p = degree of q: $y = \frac{\text{lead coeff of } p(x)}{\text{lead coeff of } q(x)}$

SLANT ASYMPTOTES:
$y = mx+b$; the quotient of $\frac{p(x)}{q(x)}$, ignoring the remainder

HOLES:
Common factors in p(x) and q(x)

EXAMPLES

1 $f(x) = \frac{3x+2}{x+2}$

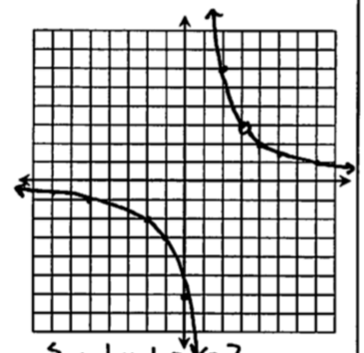


D: $\{x \mid x \neq -2\}$ R: $\{y \mid y \neq 1\}$
x-int(s): $(-2/3, 0)$ y-int: $(0,1)$
VA: $x = -2$ HA: $y=3$
SA: None Hole(s): None

2 $f(x) = \frac{6x-18}{x^2-4x+3}$

$= \frac{6(x-3)}{(x-1)(x-3)}$

$= \frac{6}{x-1}$

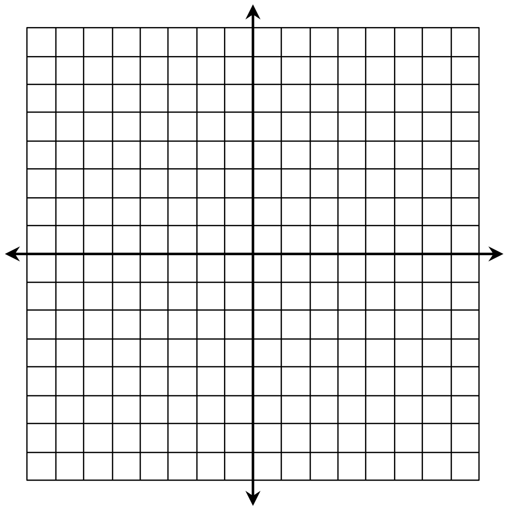


D: $\{x \mid x \neq 1, 3\}$ R: $\{y \mid y \neq 0, 3\}$
x-int(s): None y-int: $(0,-6)$
VA: $x=1$ HA: $y=0$
SA: None Hole(s): $(3,3)$

Graphing Rational Equations and Identifying Asymptotes Homework

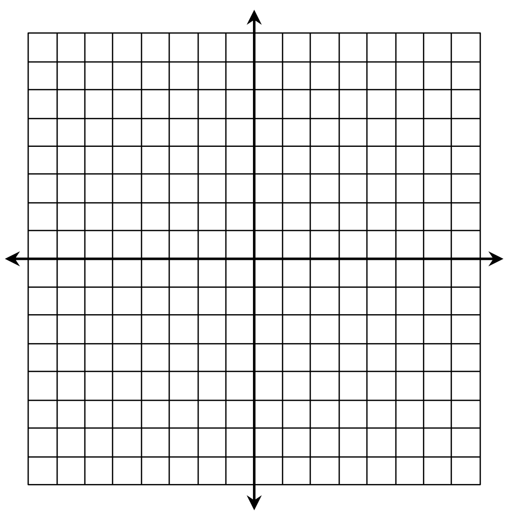
Directions: Graph each function and identify its key characteristics.

1. $f(x) = \dfrac{2x+3}{x-3}$



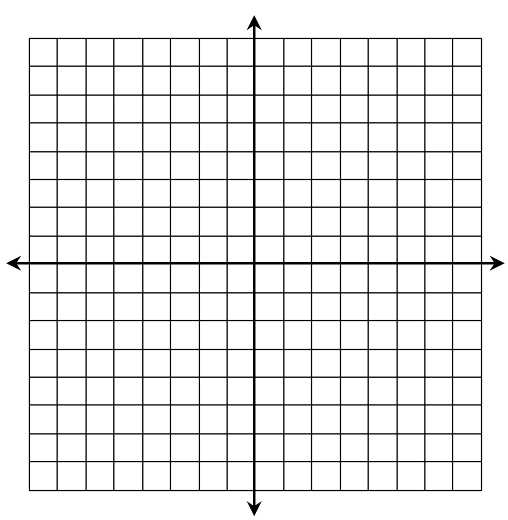
Domain:	
Range:	
x**-int(s):**	
y**-int:**	
VA:	**HA:**
SA:	
Holes:	

2. $f(x) = \dfrac{10x+20}{x^2+6x+8}$



Domain:	
Range:	
x**-int(s):**	
y**-int:**	
VA:	**HA:**
SA:	
Holes:	

3. $f(x) = \dfrac{x^2-7x+12}{x-2}$

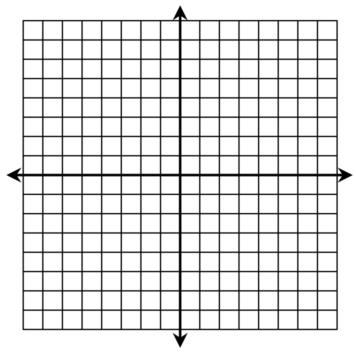


Domain:	
Range:	
x**-int(s):**	
y**-int:**	
VA:	**HA:**
SA:	
Holes:	

Graphing Rational Eq^{n}s and Asymptotes Homework Continued

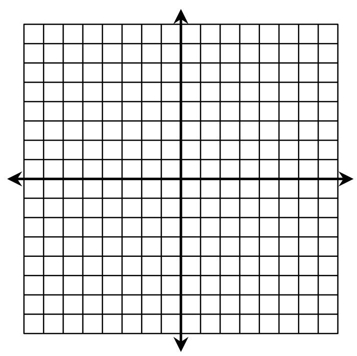
Directions: Graph each function and identify its domain, range, intercepts, vertical and horizontal asymptotes, and holes.

1. $f(x) = \dfrac{2x-3}{x+1}$



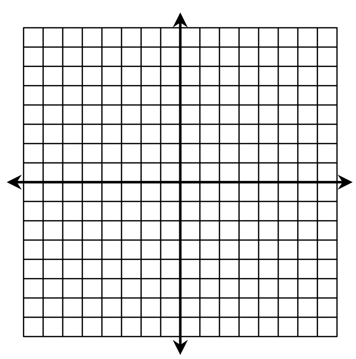
Domain:	
Range:	
***x*-int(s):**	
***y*-int:**	
VA:	
HA:	
Hole(s):	

2. $f(x) = \dfrac{3x+6}{x}$



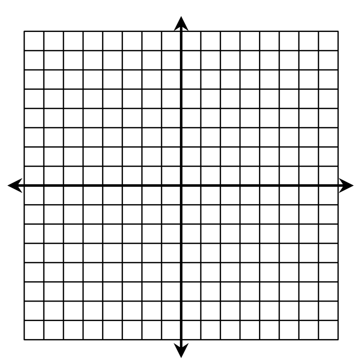
Domain:	
Range:	
***x*-int(s):**	
***y*-int:**	
VA:	
HA:	
Hole(s):	

3. $f(x) = \dfrac{x^2-4x-12}{x+2}$

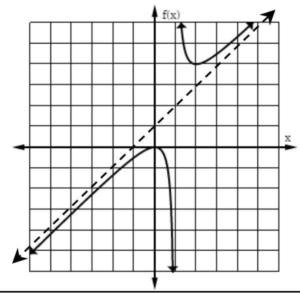


Domain:	
Range:	
***x*-int(s):**	
***y*-int:**	
VA:	
HA:	
Hole(s):	

4. $f(x) = \dfrac{x^2+3x}{x^2+5x+6}$



Domain:	
Range:	
***x*-int(s):**	
***y*-int:**	
VA:	
HA:	
Hole(s):	

Oblique (or Slant) **ASYMPTOTES**	When the ________________ of __________ is _________________ ____________ ______________ ______________ the ________________ of _________, the graph with have a slant, or **oblique asymptote**.	

The Equation of the Oblique **ASYMPTOTE**	The equation of the oblique asymptote, $y = mx + b$, is the quotient of $\frac{p(x)}{q(x)}$, ignoring the remainder.	
	Steps to find the equation of the oblique asymptote:	
	❶	Use long or synthetic division to divide $p(x)$ by $q(x)$.
	❷	Write the equation of the oblique asymptote using the quotient, ignoring the remainder.

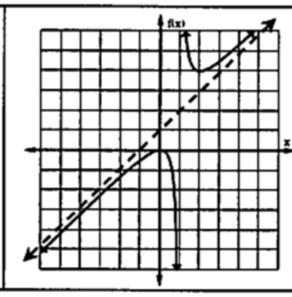
You find it.

Directions: Find the equation of the oblique asymptote.

1. $f(x)=\frac{x^2-x-14}{x+4}$ $\begin{array}{r|rrr} -4 & 1 & -1 & -14 \\ & \downarrow & -4 & 20 \\ \hline & 1 & -5 & \cancel{6} \end{array}$ $\boxed{y = x-5}$	**2.** $f(x)=\frac{3x^2-7x}{x-3}$
3. $f(x)=\frac{6x^2+4x-11}{3x+2}$	**4.** $f(x)=\frac{x^3+3x^2}{x^2+2x-3}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Oblique (or Slant) **ASYMPTOTES**	When the degree of P(x) is exactly one more than the degree of Q(x), the graph with have a slant, or **oblique asymptote**.



The Equation of the Oblique ASYMPTOTE

The equation of the oblique asymptote, $y = mx + b$, is the quotient of $\frac{p(x)}{q(x)}$, ignoring the remainder.

Steps to find the equation of the oblique asymptote:

❶	Use long or synthetic division to divide $p(x)$ by $q(x)$.
❷	Write the equation of the oblique asymptote using the quotient, ignoring the remainder.

Directions: Find the equation of the oblique asymptote.

1. $f(x) = \dfrac{x^2 - x - 14}{x + 4}$

$$\begin{array}{r|rrr} -4 & 1 & -1 & -14 \\ & \downarrow & -4 & 20 \\ \hline & 1 & -5 & \cancel{6} \end{array}$$

$$\boxed{y = x - 5}$$

2. $f(x) = \dfrac{3x^2 - 7x}{x - 3}$

$$\begin{array}{r|rrr} 3 & 3 & -7 & 0 \\ & \downarrow & 9 & 6 \\ \hline & 3 & 2 & \cancel{6} \end{array}$$

$$\boxed{y = 3x + 2}$$

3. $f(x) = \dfrac{6x^2 + 4x - 11}{3x + 2}$

$$\begin{array}{r} 2x + 0 \\ 3x+2 \overline{\big)\, 6x^2 + 4x - 11} \\ -(6x^2 + 4x) \\ \hline 0x - 11 \\ -(0x + 0) \\ \hline \cancel{-11} \end{array}$$

$$\boxed{y = 2x}$$

4. $f(x) = \dfrac{x^3 + 3x^2}{x^2 + 2x - 3}$

$$\begin{array}{r} x + 1 \\ x^2+2x-3 \overline{\big)\, x^3 + 3x^2 + 0x + 0} \\ -(x^3 + 2x^2 - 3x) \\ \hline x^2 + 3x + 0 \\ -(x^2 + 2x - 3) \\ \hline \cancel{x + 3} \end{array}$$

$$\boxed{y = x + 1}$$

Polynomial and Rational Inequalities

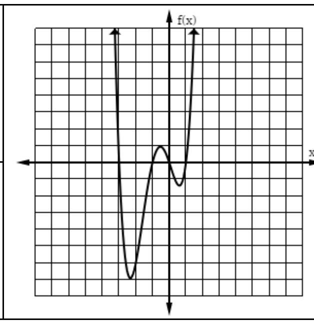
POLYNOMIAL INEQUALITY	• Given a polynomial function $f(x)$, a **polynomial inequality** has the general form $f(x) > 0$, $f(x) \geq 0$, $f(x) \neq 0$, $f(x) < 0$, or $f(x) \leq 0$. • The inequality $\boldsymbol{f(x) > 0}$ is true when ________________________________. • The inequality $\boldsymbol{f(x) < 0}$ is true when ________________________________.

LOOKING AT A GRAPH

Look at the function $f(x) = x^4 + 3x^3 - x^2 - 3x$ graphed below.

a) Name all intervals for which $f(x) > 0$.

b) Name all intervals for which $f(x) < 0$.

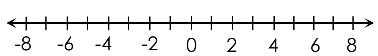


Steps to Solve a POLYNOMIAL INEQUALITY

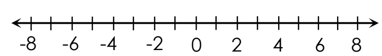
❶	Move all terms to one side of the inequality so 0 is on the other side.
❷	Completely factor the polynomial and find the zeros.
❸	Plot the zeros on a number line.
❹	Choose test points in each interval. Substitute the test points into the function to determine whether the interval is positive or negative.
❺	Write the solution using interval notation. Use parentheses or brackets where necessary.

Directions: Solve each inequality. Use the number line provided to test intervals.

1. $x^2 + 5x - 6 > 0$

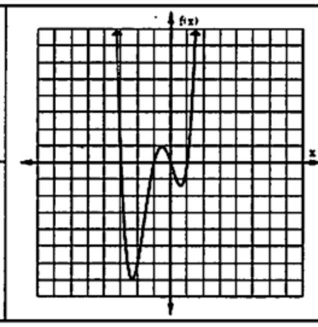


2. $2x^2 - x - 15 < 0$



Name:	Date:
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
POLYNOMIAL INEQUALITY	• Given a polynomial function $f(x)$, a **polynomial inequality** has the general form $f(x) > 0$, $f(x) \ge 0$, $f(x) \ne 0$, $f(x) < 0$, or $f(x) \le 0$. • The inequality $f(x) > 0$ is true when f(x) is positive. • The inequality $f(x) < 0$ is true when f(x) is negative.
LOOKING AT A GRAPH	**Look at the function $f(x) = x^4 + 3x^3 - x^2 - 3x$ graphed below.** **a)** Name all intervals for which $f(x) > 0$. $(-\infty, -3)$, $(-1, 0)$, $(1, \infty)$ **b)** Name all intervals for which $f(x) < 0$. $(-3, -1)$, $(0, 1)$



Steps to Solve a POLYNOMIAL INEQUALITY		
	1	Move all terms to one side of the inequality so 0 is on the other side.
	2	Completely factor the polynomial and find the zeros.
	3	Plot the zeros on a number line.
	4	Choose test points in each interval. Substitute the test points into the function to determine whether the interval is positive or negative.
	5	Write the solution using interval notation. Use parentheses or brackets where necessary.

Directions: Solve each inequality. Use the number line provided to test intervals.

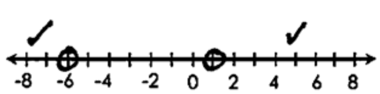
1. $x^2 + 5x - 6 > 0$

$(x+6)(x-1) > 0$ Zeros: $x = -6, 1$

-7: $(-7+6)(-7-1) > 0$
$8 > 0$ ✓

0: $(0+6)(0-1) > 0$
$-6 > 0$ X

2: $(2+6)(2-1) > 0$
$8 > 0$ ✓



$\boxed{(-\infty, -6) \cup (1, \infty)}$

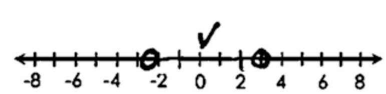
2. $2x^2 - x - 15 < 0$

$(2x+5)(x-3) < 0$ Zeros: $x = -\frac{5}{2}, 3$

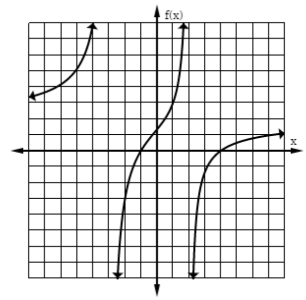
-3: $(-6+5)(-3-3) < 0$
$6 < 0$ X

0: $(0+5)(0-3) < 0$
$-15 < 0$ ✓

4: $(4+5)(4-3) < 0$
$9 < 0$ X

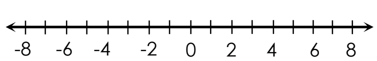


$\boxed{(-\frac{5}{2}, 3)}$

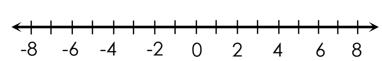
Main Ideas/Questions	Notes/Examples
Rational Inequality	• Given a rational function $f(x)$, a **rational inequality** has the general form $f(x)>0$, $f(x)\ge 0$, $f(x)\ne 0$, $f(x)<0$, or $f(x)\le 0$. • The inequality $\boldsymbol{f(x)>0}$ is true when ______________________________. • The inequality $\boldsymbol{f(x)<0}$ is true when ______________________________.
Looking at a Graph	**Look at the function $f(x)=\dfrac{2x^2-6x-8}{x^2+x-6}$ graphed below.** **a)** Name all intervals for which $f(x)>0$. **b)** Name all intervals for which $f(x)<0$.  Notice that a rational function switches signs at both its **zeros** and its **vertical asymptotes**!
Steps to Solve a Rational Inequality	❶ Move all terms to one side of the inequality so 0 is on the other side. ❷ Find the zeros of both the numerator and the denominator by factoring. ❸ Plot these points on a number line. **(Asymptotes ALWAYS get an open circle!)** ❹ Choose test points in each interval. Substitute the test points into the function to determine whether the interval is positive or negative. ❺ Write the solution using interval notation. Use parentheses or brackets where necessary.

Directions: Solve each inequality. Use the number line provided to test intervals.

1. $\dfrac{x+5}{x+2}>0$



2. $\dfrac{x}{x^2-5x-6}<0$



Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
Rational Inequality	• Given a rational function $f(x)$, a **rational inequality** has the general form $f(x)>0$, $f(x)\ge 0$, $f(x)\ne 0$, $f(x)<0$, or $f(x)\le 0$. • The inequality $f(x)>0$ is true when f(x) is positive. • The inequality $f(x)<0$ is true when f(x) is negative.

Looking at a Graph

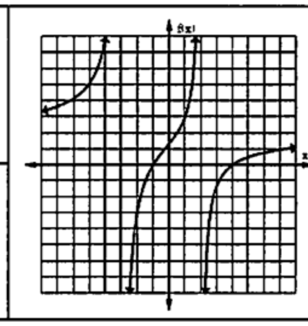
Look at the function $f(x)=\dfrac{2x^2-6x-8}{x^2+x-6}$ graphed below.

a) Name all intervals for which $f(x)>0$.

$(-\infty,-3)$, $(-1,2)$, $(4,\infty)$

b) Name all intervals for which $f(x)<0$.

$(-3,-1)$, $(2,4)$



Notice that a rational function switches signs at both its **zeros** and its **vertical asymptotes**!

Steps to Solve a Rational Inequality

1	Move all terms to one side of the inequality so 0 is on the other side.
2	Find the zeros of both the numerator and the denominator by factoring.
3	Plot these points on a number line. **(Asymptotes ALWAYS get an open circle!)**
4	Choose test points in each interval. Substitute the test points into the function to determine whether the interval is positive or negative.
5	Write the solution using interval notation. Use parentheses or brackets where necessary.

Directions: Solve each inequality. Use the number line provided to test intervals.

1. $\dfrac{x+5}{x+2}>0$

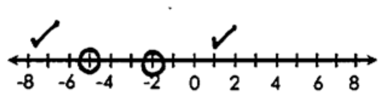
Zero: $x=-5$

Asym: $x=-2$

-6: $\frac{-1}{-4}>0$; $\frac{1}{4}>0$ ✓

-3: $\frac{2}{-1}>0$; $-2>0$ X

0: $\frac{5}{2}>0$ ✓



$\boxed{(-\infty,-5)\cup(-2,\infty)}$

2. $\dfrac{x}{x^2-5x-6}<0$

Zero: $x=0$

Asym: $(x-6)(x+1)$

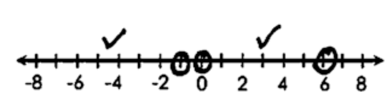
$x=6$, $x=-1$

-2: $\frac{-2}{8}<0$; $-\frac{1}{4}<0$ ✓

-0.5: $\frac{-.5}{-3.25}<0$; $\frac{2}{13}<0$ X

1: $\frac{1}{-10}<0$ ✓

7: $\frac{7}{8}<0$ X



$\boxed{(-\infty,-1)\cup(0,6)}$

Topic #2: Discriminant of a Quadratic Equation

Given a quadratic equation of the form $ax^2 + bx + c = 0$, you can determine the number and type of roots (solutions) by finding the discriminant of the equation.			
Discriminant Formula	**Value of d**	**# of Roots**	**Type of Roots**
$b^2 - 4ac$	$d > 0$ (a perfect square)	2	real + rational
	$d > 0$ (NOT a perfect square)	2	real + irrational
	$d = 0$	1	real + rational
	$d < 0$	2	complex/ imaginary

Find the discriminant of each equation, then determine the number and type of roots.

9. $x^2 + 12x - 27 = 0$

$(12)^2 - 4(1)(-27)$

$144 + 108 = 252$

$\boxed{\text{2 irrational roots}}$

10. $9x^2 + 30x + 25 = 0$

$(30)^2 - 4(9)(25)$

$900 - 900 = 0$

$\boxed{\text{1 rational root}}$

11. $2x^2 + 11 = 1 \rightarrow 2x^2 + 10 = 0$

$(0)^2 - 4(2)(10)$

$0 - 80 = -80$

$\boxed{\text{2 imaginary roots}}$

12. $5x^2 + 5 = x^2 - 12x \rightarrow 4x^2 + 12x + 5 = 0$

$(12)^2 - 4(4)(5)$

$144 - 80 = 64$

$\boxed{\text{2 rational roots}}$

Topic #3: Solving Quadratic Equations

Methods for Solving Quadratic Equations:
Factoring, Square Roots, Completing the Square, The Quadratic Formula

Solve using the most appropriate method. Simplify all irrational and complex solutions.

13. $x^2 - 9x - 43 = 27 \rightarrow x^2 - 9x - 70 = 0$

$(x-14)(x+5) = 0$

$x-14=0$	$x+5=0$
$x=14$	$x=-5$

$\boxed{x = \{-5, 14\}}$

14. $x^2 - 8x + 33 = 5 \rightarrow x^2 - 8x + 28 = 0$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(28)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 - 112}}{2}$$

$$= \frac{8 \pm \sqrt{-48}}{2}$$

$$= \frac{8 \pm 4i\sqrt{3}}{2} = \boxed{4 \pm 2i\sqrt{3}}$$