Sequences

Arithmetic

$a_2 - a_1 = a_3 - a_2 = \text{difference}$

$a_n = a_1 + d(n-1)$

Geometric

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \text{ratio}$

$a_n = a_1 \cdot r^{(n-1)}$

Series

Summary Notation

$$\sum_{\text{first}}^{\text{last}} \text{formula}$$

Arithmetic Series

$a_n = a_1 + d^{(n-1)}$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$

Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Infinite Geometric

$|r| < 1$, then convergent

$|r| > 1$, then divergent

Convergent Infinite Geometric Series

$$S_n = a_1\left(\frac{1}{1-r}\right)$$

STEM-ersion

w/ Arithmetic Sequences

Name ____________________

Date ____________________

Wedding Planner

Ida is a wedding planner. She is renting tables for a wedding. The wedding requires a special rectangular table for the head table. The RSVP list is not yet set, so she needs to give the couple a range of prices for the table rentals. The chart below shows the different rental options and their associated costs. The couple estimates between 80 and 100 people will attend the wedding. Ida wants to give them the best recommendation as well as an equation they can use to calculate the cost based on the number of people who are attending.

Option	A	B	C	D
Cost: Head Table	\$50	\$34	\$25	\$42
Cost per table	\$13	\$20	\$28	\$34
#of people table seats	4	6	8	10

Evidence

Use this space to make any calculations and show work.

Option	A	B	C	D
Equation	$c = 50 + 13(n - 1)$ or $c = 37 + 13n$	$c = 34 + 20(n - 1)$ or $c = 14 + 20n$	$c = 25 + 28(n - 1)$ or $c = -3 + 25n$	$c = 42 + 34(n - 1)$ or $c = 8 + 34n$
Cost at 80 people	21 tables rented \$310	15 tables rented \$314	11 tables rented \$305	9 tables rented \$314
Cost at 100 people	26 tables rented \$375	18 tables rented \$374	14 tables rented \$389	11 tables rented \$382

Analysis of the evidence

Interpret the Evidence. What does it mean?

The student can compare the price of renting enough tables for 80 and 100 people to determine which is the best deal to go with.

Conclusion or Recommendation

The student can select any of the options based on their analysis of the cost between 80 and 100 people.

STEM-ersion

w/ Geometric Series

Name ____________________

Date ____________________

ANSWER KEY

Ki[illegible] d [illegible] ur [illegible] ns [illegible] o [illegible] wn the [illegible] cts [illegible] sa[illegible] gs [illegible] e [illegible] o [illegible] ch [illegible] no [illegible] by co[illegible] he [illegible] tim[illegible] akes to create a colony of the bacteria to how long it takes the bacteria to break down the byproducts to 0.1 ppm. Which strain should Kimberly market?

Threshold Colony				
Initial amount	10 bacteria	8 bacteria	4 bacteria	12 bacteria
Growth rate	Doubles every hour	Quadruples every 3 hours	Quadruples every hour	8 times every 7 hours
Threshold amount	10,000	10,000	10,000	10,000
Byproduct Breakdown				
Initial amount	$9.57\times10^{30}\ ppm$	$9.57\times10^{30}\ ppm$	$9.57\times10^{30}\ ppm$	$9.57\times10^{30}\ ppm$
Breakdown rate	30% per hour	44% per hour	18% per hour	60% per hour

Evidence

Use this space to make any calculations and show work.

Bacteria	Alpha	Beta	Gamma	Delta
Time to reach threshold numbers	10.0 hours	24.8 hours	5.64 hours	50.4 hours
Time to complete breakdown	207 hours or 8.6 days	127 hours or 5.3 days	371 hours or 15.5 days	80.4 hours or 3.4 days

Analysis of the evidence

Interpret the Evidence. What does it mean?

The student can compare the time to reach threshold numbers with the time to complete breakdown to determine which bacteria should be marketed.

Conclusion or Recommendation

The student can choose any of the bacteria depending on if they value the speed of breakdown or of reaching the threshold colony more.

Fill-in-the-answer questions for SEQUENCES & SERIES

For each sequence (a) determine whether it is arithmetic or geometric and (b) write an explicit rule for the n^{th} term.

1. $\{-8, -2, 4, 10, \ldots\}$

2. $\{27, -18, 12, -8, \ldots\}$

3. $\left\{-\frac{3}{8}, -\frac{3}{2}, -6, -24, \ldots\right\}$

4. $\left\{-\frac{11}{6}, -\frac{37}{12}, -\frac{13}{3}, -\frac{67}{12} \ldots\right\}$

1. a) ________________

b) ________________

2. a) ________________

b) ________________

3. a) ________________

b) ________________

4. a) ________________

b) ________________

For each series, (a) determine whether it is arithmetic or geometric, then (b) find the indicated sum, if possible.

5. $\{-2 + 8 - 32 + 128 + \ldots\};\ S_{11}$

6. $\left\{-\frac{1}{6} + \frac{4}{3} + \frac{17}{6} + \frac{13}{3} + \ldots\right\}; S_{16}$

7. $\sum_{n=1}^{9} -4 \cdot \left(-\frac{3}{2}\right)^{n-1}$

8. $\sum_{c=1}^{24} (155 - 3c)$

5. a) ________________

b) ________________

6. a) ________________

b) ________________

7. a) ________________

b) ________________

8. a) ________________

b) ________________

Fill-in-the-answer questions for SEQUENCES & SERIES

For each sequence (a) determine whether it is arithmetic or geometric and (b) write an explicit rule for the n^{th} term.

1. $\{-8, -2, 4, 10, \ldots\}$

$a_n = 6(n-1) - 8$

$= 6n - 6 - 8$

$= 6n - 14$

2. $\{27, -18, 12, -8, \ldots\}$

$r = -2/3$

3. $\left\{-\frac{3}{8}, -\frac{3}{2}, -6, -24, \ldots\right\}$

$r = 4$

4. $\left\{-\frac{11}{6}, -\frac{37}{12}, -\frac{13}{3}, -\frac{67}{12} \ldots\right\}$

$a_n = -\frac{5}{4}(n-1) - \frac{11}{6}$

$= -\frac{5}{4}n + \frac{5}{4} - \frac{11}{6}$

$= -\frac{5}{4}n - \frac{7}{12}$

1. a) arithmetic

b) $a_n = 6n - 14$

2. a) geometric

b) $a_n = 27\left(-\frac{2}{3}\right)^{n-1}$

3. a) geometric

b) $a_n = -\frac{3}{8} \cdot 4^{n-1}$

4. a) arithmetic

b) $a_n = -\frac{5}{4}n - \frac{7}{12}$

For each series, (a) determine whether it is arithmetic or geometric, then (b) find the indicated sum, if possible.

5. $\{-2 + 8 - 32 + 128 + \ldots\}; S_{11}$

$S_{11} = \frac{-2(1-(-4)^{11})}{1-(-4)}$

$= -1,677,722$

6. $\left\{-\frac{1}{6} + \frac{4}{3} + \frac{17}{6} + \frac{13}{3} + \ldots\right\}; S_{16}$

$S_{16} = 16\left(\frac{-\frac{1}{6} + \frac{67}{3}}{2}\right)$

$= \frac{532}{3}$

$a_n = a_1 + d^{(n-1)}$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$

7. $\sum_{n=1}^{9} -4 \cdot \left(-\frac{3}{2}\right)^{n-1}$

$S_9 = \frac{-4(1-(-3/2)^9)}{1-(-3/2)}$

$= -\frac{4039}{64}$

8. $\sum_{c=1}^{24} (155 - 3c)$

$S_{24} = 24\left(\frac{152 + 83}{2}\right)$

$= 2,820$

5. a) geometric

b) $-1,677,722$

6. a) arithmetic

b) $\frac{532}{3}$ 177 .3

7. a) geometric

b) $-\frac{4039}{64}$ 63.11

8. a) arithmetic

b) 2,820

9. $\sum_{k=2}^{46}\left(\frac{2}{3}k+\frac{5}{6}\right)$

10. $\sum_{i=3}^{10}\left(-\frac{5}{6}\right)\cdot 3^{i-1}$

11. $\sum_{m=1}^{\infty}\frac{1}{3}\cdot 4^{m-1}$

12. $\sum_{p=1}^{\infty}125\cdot\left(-\frac{1}{5}\right)^{p-1}$

9. a) ____________

b) ____________

10. a) ____________

b) ____________

11. a) ____________

b) ____________

12. a) ____________

b) ____________

Solve each problem using a sequence or series formula.

13. The florist got a new helium tank with 300 cubic feet of helium. On the first day, 0.8 cubic feet of helium was used to fill balloons. Each day thereafter, 25% more helium was used than the day prior. How many days until the tank is empty?

14. Brad got a job with a starting wage of $9.25 per hour. He gets an annual raise of $0.80 per hour. After many years will Brad reach a wage of at least $20 per hour?

13. ____________

14. ____________

15. ____________

16. ____________

15. Caryn got a new car. The table to the right gives the number of miles she put on the car in each of the first three years that she owned it. If this pattern continues and she keeps the car for 12 years, how many total miles will be on the car?

Year	Miles
1	11,400
2	12,050
3	12,700

16. In 2015, the deer population in a certain area was recorded at 1,200. Since then, the population has increased by about 9% each year. In which year will the deer population reach 3,000?

Since the first # in the series is 3, the tenth would actually be S_8

9. $\sum_{k=2}^{46}\left(\frac{2}{3}k+\frac{5}{6}\right)$

$S_{45} = 45\left(\frac{\frac{13}{6}+\frac{63}{2}}{2}\right)$

$S_n = n\left(\frac{a_1+a_n}{2}\right) = 757.5 \quad \left(\frac{1515}{2}\right)$

10. $\sum_{i=3}^{10}\left(-\frac{5}{6}\right)\cdot 3^{i-1}$ $\quad S_n = \frac{a_1(1-r^n)}{1-r}$

$S_8 = \frac{-\frac{15}{2}(1-3^8)}{1-3}$

$= -24{,}600$

11. $\sum_{m=1}^{\infty}\frac{1}{3}\cdot 4^{m-1}$

No sum (divergent)

12. $\sum_{p=1}^{\infty} 125\cdot\left(-\frac{1}{5}\right)^{p-1}$

$S_n = 125\left(\frac{1}{1-(-1/5)}\right)$

$= \frac{625}{6}$

9. a) arithmetic

b) 757.5 774.3

10. a) geometric

b) -24,600 -221430

11. a) geometric

b) no sum (divergent)

12. a) geometric

b) $\frac{625}{6}$

Solve each problem using a sequence or series formula.

13. The florist got a new helium tank with 300 cubic feet of helium. On the first day, 0.8 cubic feet of helium was used to fill balloons. Each day thereafter, 25% more helium was used than the day prior. How many days until the tank is empty?

$300 = \frac{.8(1-1.25^n)}{1-1.25}$

$-75 = .8(1-1.25^n)$

$-93.75 = 1-1.25^n$

$-94.75 = -1.25^n$

$\log(94.75) = n\cdot\log(1.25)$

$n = 20.4 \rightarrow 21$ days

14. Brad got a job with a starting wage of \$9.25 per hour. He gets an annual raise of \$0.80 per hour. After many years will Brad reach a wage of at least \$20 per hour?

$20 = .8(n-1)+9.25$

$10.75 = .8(n-1)$

$13.44 = n-1$

$14.44 = n \rightarrow 15$ years

15. Caryn got a new car. The table to the right gives the number of miles she put on the car in each of the first three years that she owned it. If this pattern continues and she keeps the car for 12 years, how many total miles will be on the car?

Year	Miles
1	11,400
2	12,050
3	12,700

$d = 650$

$S_{12} = 12\left(\frac{11400+18550}{2}\right)$

$= 179{,}700$

16. In 2015, the deer population in a certain area was recorded at 1,200. Since then, the population has increased by about 9% each year. In which year will the deer population reach 3,000?

$3000 = 1200(1.09)^{n-1}$

$2.5 = 1.09^{n-1}$

$\log 2.5 = n-1(\log 1.09)$

$10.63 = n-1$

$11.63 = n \rightarrow 2026$

13. 21 days

14. 15 years

15. 179,700

16. 2026

Real-life word problems for SEQUENCES & SERIES

SEQUENCE Applications

1. A library book that is one day late is charged a $1.95 fee. Each day thereafter, it is charged an extra $0.20. Find the fee for a book that is 35 days late.

2. Tucker took an 800-milligram dose of medicine for his headache. The table to the left shows the amount of medicine remaining in his bloodstream after each of the first three hours. After how many hours will the amount of medicine reach 50 milligrams?

Hour	Milligrams
1	800
2	680
3	578

SERIES Applications

3. Stocks at a company were initiallly issued at $9.80 per share. The value of the shares has increased by 25% each year. If Ari bought 20 shares each year since they were issued, find her total investment after 15 years.

4. Evan got a job with a starting salary of $36,000, with a $1,500 raise each subsequent year. How many years will it take for his total earnings to reach $1,000,000?

MIXED Applications

5. A ball is dropped from a tower. The table below shows the height of the ball after each of the first three bounces. Find the height of the ball after the 12^{th} bounce.

Bounce	Height (ft)
1	50
2	45
3	40.5

Real-life word problems for SEQUENCES & SERIES

Main Ideas/Questions	Notes/Examples

SEQUENCE Applications

1. A library book that is one day late is charged a $1.95 fee. Each day thereafter, it is charged an extra $0.20. Find the fee for a book that is 35 days late.

$a_n = 0.2(n-1) + 1.95$
$= 0.2n + 1.75$

$a_{25} = 0.2(25) + 1.75$
$= \boxed{\$8.75}$

2. Tucker took an 800-milligram dose of medicine for his headache. The table to the left shows the amount of medicine remaining in his bloodstream after each of the first three hours. After how many hours will the amount of medicine reach 50 milligrams?

Hour	Milligrams
1	800
2	680
3	578

$a_n = 800(0.85)^{n-1}$
$50 = 800(.85)^{n-1}$
$0.0625 = .85^{n-1}$

$\log(0.0625) = (n-1)\log(.85)$
$17.06 = n-1$
$18.06 = n$

$\boxed{18.06 \text{ hr}}$

SERIES Applications

3. Stocks at a company were initially issued at $9.80 per share. The value of the shares has increased by 25% each year. If Ari bought 20 shares each year since they were issued, find her total investment after 15 years.

$S_{15} = \dfrac{196(1-1.25^{15})}{1-1.25}$

$= \boxed{\$21,498.62}$

4. Evan got a job with a starting salary of $36,000, with a $1,500 raise each subsequent year. How many years will it take for his total earnings to reach $1,000,000?

$a_n = 1500(n-1) + 36000$

$a_n = 1500n + 34500$

$1000000 = n\left(\dfrac{36000 + 1500n + 34500}{2}\right)$

$2000000 = n(1500n + 70500)$

$2000000 = 1500n^2 + 70500n$

$1500n^2 + 70500n - 2000000 = 0$

$3n^2 + 141n - 4000 = 0$

$n = \dfrac{-141 \pm \sqrt{141^2 - 4(3)(-4000)}}{2(3)}$

$n = 19.92, ~~-66.92~~$

$\boxed{20 \text{ years}}$

MIXED Applications

5. A ball is dropped from a tower. The table below shows the height of the ball after each of the first three bounces. Find the height of the ball after the 12th bounce.

Bounce	Height (ft)
1	50
2	45
3	40.5

$a_n = 50(0.9)^{n-1}$

$a_{12} = 50(0.9)^{11}$
$= \boxed{15.69 \text{ ft}}$

6. Logs are stacked so that they are 40 logs on the bottom row and each row thereafter has 2 logs fewer than the row below it. If the top row has 8 logs, find the total number of logs in the stack.

7. When Michelle brought her newborn son John home, he slept just three hours the first night. Each night thereafter, he slept an extra 5 minutes than the previous night. How many nights will it take John to sleep an 8-hour stretch?

8. Elijah started a new Instagram account and gained 8 new followers in his first week. Each subsequent week, he gained twice as many new followers than he did the previous week. How many total followers does Elijah have after 16 weeks?

9. There are 20 seats in the first row of a concert hall. Each row thereafter has 3 seats more than the previous row. If 600 students are coming to the hall for a field trip, how many rows will be needed, assuming they are seated starting with the first row?

10. The table to the left shows the value of a car that was manufactured in 2012, along with its value for three subsequent years. In what year will the value of the car reach $4,000?

Year	Value
2012	$37,500
2013	$31,500
2014	$26,460
2015	$22,226.40

6. Logs are stacked so that they are 40 logs on the bottom row and each row thereafter has 2 logs fewer than the row below it. If the top row has 8 logs, find the total number of logs in the stack.

$a_n = -2(n-1) + 40$

$a_n = -2n + 42$

$8 = -2n + 42$

$-34 = -2n$

$n = 17$

$S_n = 17\left(\frac{40+8}{2}\right) = \boxed{408 \text{ logs}}$

7. When Michelle brought her newborn son John home, he slept just three hours the first night. Each night thereafter, he slept an extra 5 minutes than the previous night. How many nights will it take John to sleep an 8-hour stretch?

$a_n = 5(n-1) + 180$

$a_n = 5n + 175$

$480 = 5 + 175$

$305 = 5n$

$n = 61$

$\boxed{61 \text{ days}}$

8. Elijah started a new Instagram account and gained 8 new followers in his first week. Each subsequent week, he gained twice as many new followers than he did the previous week. How many total followers does Elijah have after 16 weeks?

$a_n = 8(2)^{n-1}$

$a_{16} = 8(2)^{15} = \boxed{262{,}144}$

9. There are 20 seats in the first row of a concert hall. Each row thereafter has 3 seats more than the previous row. If 600 students are coming to the hall for a field trip, how many rows will be needed, assuming they are seated starting with the first row?

$a_n = 3(n-1) + 20$

$a_n = 3n + 17$

$600 = n\left(\frac{20 + 3n + 17}{2}\right)$

$1200 = n(3n + 37)$

$3n^2 + 37n - 1200 = 0$

$n = \frac{-37 \pm \sqrt{37^2 - 4(3)(-1200)}}{2(3)}$

$n = 14.76, -27.$

$\boxed{15 \text{ rows}}$

10. The table to the left shows the value of a car that was manufactured in 2012, along with its value for three subsequent years. In what year will the value of the car reach $4,000?

Year	Value
2012	$37,500
2013	$31,500
2014	$26,460
2015	$22,226.40

$a_n = 37500(.84)^{n-1}$

$4000 = 37500(.84)^{n-1}$

$\frac{8}{75} = (.84)^{n-1}$

$\log(8/75) = (n-1)\log(.84)$

$12.84 = n-1$

$13.84 = n$

13 years $\rightarrow$ $\boxed{2024}$