Sequences

Arithmetic

$a_2 - a_1 = a_3 - a_2 = \text{difference}$ $\quad a_n = a_1 + d(n-1)$

Geometric

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \text{ratio}$ $\quad a_n = a_1 \cdot r^{n-1}$

Series

Summary Notation

$$\sum_{\text{first}}^{\text{last}} \text{formula}$$

Arithmetic Series

$a_n = a_1 + d(n-1)$ $\quad S_n = n\left(\frac{a_1 + a_n}{2}\right)$

Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Infinite Geometric

$|r| < 1$, then convergent

$|r| > 1$, then divergent

Convergent Infinite Geometric Series

$$S_n = a_1\left(\frac{1}{1-r}\right)$$

Main Ideas/Questions	Notes/Examples

SERIES

Sequence			
Series			

PARTIAL SUMS

Directions: Find the partial sum for each given sequence.

1. $\{2, 6, 10, 14, 18, \ldots\}$; find S_7	**2.** $\{1, -2, 4, -8, 16, \ldots\}$; find S_8
3. $\{1, 1, 2, 3, 5, 8, \ldots\}$; find S_{10}	**4.** $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\right\}$; find S_3

SUMMATION NOTATION

A way to represent a series using the greek letter Σ to denote the sum.

$$\sum_{n=1}^{5} 2n$$

Find the sum of the series above:

EXAMPLES

Directions: Expand each series and evaluate.

9. $\sum_{n=1}^{14} (n+5)$	**10.** $\sum_{n=1}^{11} (-12n)$

Main Ideas/Questions	Notes/Examples
Arithmetic Series (when you ± a common difference to get the next term)	To find the sum of an **arithmetic series**, use the following formula: where ***n*** is the ______________________________________, a_1 is the _______________________, and a_n is the ________________________.
Here are some examples.	**Directions:** Find the indicated sum for each arithmetic series. **1.** $\{7+10+13+16+\ldots\}$; S_{18} **2.** $\{50+42+34+26+\ldots\}$; S_{35}

How does this pertain to me!

Gideon has decided to train for a marathon. He ran 2.4 miles the first day, 2.55 miles the second day, 2.7 miles on the third day. If this pattern continues, find the total distance he ran after 60 days.

Minute	Tickets
1	35
2	43
3	51
4	59

BTS tickets opened up for sale online. The number of people that purchased tickets in each of the first 4 minutes is shown in the table to the left. If this pattern continues, and the concert venue can hold a maximum of 75,000 people, find the number of tickets left after the first 2 hours.

Main Ideas/Questions	Notes/Examples
Geometric Series (when you multiply by a common ratio to get the next term)	The sum of a geometric sequence. To find the sum of a **geometric series**, use the following formula: where n is the number of terms, a_1 is the first term, and r is the common ratio.
Here are some examples.	**Find the indicated sum for each geometric series.** **1.** $\{2+10+50+250+\ldots\}$; S_9 **2.** $\{72+(-36)+18+(-9)+\ldots\}$; S_8
Seriously, when will I ever use this?	Gideon is saving money for a new suit. In the first month, he saves $4,000. Each month after, he saves 1.5% more than the previous month. Find the total money he will have saved in 1 year.
	The girls in Precalculus class want to be millionaires. If they each save one cent on the first day, 2 cents on the second day, 4 cents on the third day, and so on. How many days will it take them to save a million dollars?

INFINITE GEOMETRIC SERIES

Find the partial sums for each infinite series below:

$\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...\right\}$	
S_1	.5
S_2	.75
S_3	.875
S_4	.9375
S_5	.96875
S_6	.984375

$S_n \to 1$

A series that approaches a certain sum is called a **CONVERGENT SERIES.**

$\left\{\frac{1}{2}+1+2+4+8+...\right\}$	
S_1	.5
S_2	1.5
S_3	3.5
S_4	7.5
S_5	15.5
S_6	31.5

$S_n \to \infty$

A series that does not have a certain sum is called a **DIVERGENT SERIES.**

- If $|r| < 1$, then the series is convergent.
- If $|r| > 1$, then the series is divergent.

Convergent Series FORMULA

To find the sum of a **convergent infinite geometric series,** use the formula:

EXAMPLES

Determine if the series is converent or divergent. If convergent, find the sum.

11. $\{2+(-12)+72+(-432)+...\}$

12. $\left\{72+24+8+\frac{8}{3}+...\right\}$

13. $\{(-180)+90+(-45)+22.5+...\}$

14. $\left\{1+\frac{5}{4}+\frac{25}{16}+\frac{125}{64}+...\right\}$

Main Ideas/Questions	Notes/Examples	
Arithmetic vs. Geometric **SEQUENCES**	**Directions:** Determine whether the sequence is arithmetic, geometric, or neither. If arithmetic or geometric, write an explicit formula to find the n^{th} term, then find a_7.	
Arithmetic $a_2 - a_1 = a_3 - a_2 = r$ $a_n = a_1 + d(n-1)$	**1.** $\{14, -42, 126, -378, \ldots\}$ geometric; $r = -3$ $a_n = 14(-3)^{n-1}$ $a_7 = 14(-3)^6 = 10{,}206$	**2.** $\{1, -1, 2, -2, \ldots\}$ Neither
Geometric $\frac{a_2}{a_1} = \frac{a_3}{a_2} = d$ $a_n = a_1 \cdot r^{n-1}$	**3.** $\{53, 44, 35, 26, \ldots\}$ arithmetic; $d = -9$ $a_n = -9(n-1) + 53$ $a_n = -9n + 62$ $a_7 = -9(7) + 62 = -1$	**4.** $\left\{-\frac{15}{4}, -\frac{13}{4}, -\frac{11}{4}, -\frac{9}{4}, \ldots\right\}$ arithmetic; $d = \frac{1}{2}$ $a_n = \frac{1}{2}(n-1) - \frac{15}{4}$ $a_n = \frac{1}{2}n - \frac{17}{4}$ $a_7 = \frac{1}{2}(7) - \frac{17}{4} = -\frac{3}{4}$

Arithmetic vs. Geometric **SERIES**	**Directions:** Determine whether the series is arithmetic or geometric. Then find the indicated sum.	
	9. $\{18 + 25 + 32 + 39 + \ldots\}$; S_{19} arithmetic $a_1 = 18$, $a_{19} = 144$, $n = 19$ $S_{19} = 19\left(\frac{18+144}{2}\right) = 1539$	**10.** $\{3 - 15 + 75 - 375 + \ldots\}$; S_7 geometric $a_1 = 3$, $r = -5$, $n = 7$ $S_7 = \frac{3(1-(-5)^7)}{1-(-5)} = 39{,}063$

$$\sum_{first}^{last} formula$$

Directions: Determine whether the series is arithmetic or geometric, then find the sum, if possible.		
15. $\sum_{m=1}^{28}(23 - 6m)$ arithmetic $a_1 = 17$, $a_{28} = -145$, $n = 28$ $S_{28} = 28\left(\frac{17-145}{2}\right) = -1792$	**16.** $\sum_{y=1}^{13} -5 \cdot 2^{y-1}$ geometric $a_1 = -5$, $r = 2$, $n = 13$ $S_{13} = \frac{-5(1-2^{13})}{1-2} = -40{,}955$	

Fill-in-the-answer questions for SEQUENCES & SERIES

For each sequence (a) determine whether it is arithmetic or geometric and (b) write an explicit rule for the n^{th} term.

1. $\{-8, -2, 4, 10, \ldots\}$

2. $\{27, -18, 12, -8, \ldots\}$

3. $\left\{-\frac{3}{8}, -\frac{3}{2}, -6, -24, \ldots\right\}$

4. $\left\{-\frac{11}{6}, -\frac{37}{12}, -\frac{13}{3}, -\frac{67}{12} \ldots\right\}$

1. a) ________________

b) ________________

2. a) ________________

b) ________________

3. a) ________________

b) ________________

4. a) ________________

b) ________________

For each series, (a) determine whether it is arithmetic or geometric, then (b) find the indicated sum, if possible.

5. $\{-2 + 8 - 32 + 128 + \ldots\};\ S_{11}$

6. $\left\{-\frac{1}{6} + \frac{4}{3} + \frac{17}{6} + \frac{13}{3} + \ldots\right\}; S_{16}$

7. $\sum_{n=1}^{9} -4 \cdot \left(-\frac{3}{2}\right)^{n-1}$

8. $\sum_{c=1}^{24} (155 - 3c)$

5. a) ________________

b) ________________

6. a) ________________

b) ________________

7. a) ________________

b) ________________

8. a) ________________

b) ________________

9. $\sum_{k=2}^{46}\left(\frac{2}{3}k+\frac{5}{6}\right)$

10. $\sum_{i=3}^{10}\left(-\frac{5}{6}\right)\cdot 3^{i-1}$

9. a) ________________

b) ________________

10. a) ________________

b) ________________

11. $\sum_{m=1}^{\infty}\frac{1}{3}\cdot 4^{m-1}$

12. $\sum_{p=1}^{\infty}125\cdot\left(-\frac{1}{5}\right)^{p-1}$

11. a) ________________

b) ________________

12. a) ________________

b) ________________

Solve each problem using a sequence or series formula.

13. The florist got a new helium tank with 300 cubic feet of helium. On the first day, 0.8 cubic feet of helium was used to fill balloons. Each day thereafter, 25% more helium was used than the day prior. How many days until the tank is empty?

13. ______________

14. ______________

15. ______________

16. ______________

14. Brad got a job with a starting wage of $9.25 per hour. He gets an annual raise of $0.80 per hour. After many years will Brad reach a wage of at least $20 per hour?

15. Caryn got a new car. The table to the right gives the number of miles she put on the car in each of the first three years that she owned it. If this pattern continues and she keeps the car for 12 years, how many total miles will be on the car?

Year	Miles
1	11,400
2	12,050
3	12,700

16. In 2015, the deer population in a certain area was recorded at 1,200. Since then, the population has increased by about 9% each year. In which year will the deer population reach 3,000?

Real-life word problems for SEQUENCES & SERIES

SEQUENCE Applications

1. A library book that is one day late is charged a $1.95 fee. Each day thereafter, it is charged an extra $0.20. Find the fee for a book that is 35 days late.

Hour	Milligrams
1	800
2	680
3	578

2. Tucker took an 800-milligram dose of medicine for his headache. The table to the left shows the amount of medicine remaining in his bloodstream after each of the first three hours. After how many hours will the amount of medicine reach 50 milligrams?

SERIES Applications

3. Stocks at a company were initiallly issued at $9.80 per share. The value of the shares has increased by 25% each year. If Ari bought 20 shares each year since they were issued, find her total investment after 15 years.

4. Evan got a job with a starting salary of $36,000, with a $1,500 raise each subsequent year. How many years will it take for his total earnings to reach $1,000,000?

MIXED Applications

5. A ball is dropped from a tower. The table below shows the height of the ball after each of the first three bounces. Find the height of the ball after the 12th bounce.

Bounce	Height (ft)
1	50
2	45
3	40.5

6. Logs are stacked so that they are 40 logs on the bottom row and each row thereafter has 2 logs fewer than the row below it. If the top row has 8 logs, find the total number of logs in the stack.

7. When Michelle brought her newborn son John home, he slept just three hours the first night. Each night thereafter, he slept an extra 5 minutes than the previous night. How many nights will it take John to sleep an 8-hour stretch?

8. Elijah started a new Instagram account and gained 8 new followers in his first week. Each subsequent week, he gained twice as many new followers than he did the previous week. How many total followers does Elijah have after 16 weeks?

9. There are 20 seats in the first row of a concert hall. Each row thereafter has 3 seats more than the previous row. If 600 students are coming to the hall for a field trip, how many rows will be needed, assuming they are seated starting with the first row?

10. The table to the left shows the value of a car that was manufactured in 2012, along with its value for three subsequent years. In what year will the value of the car reach $4,000?

Year	**Value**
2012	$37,500
2013	$31,500
2014	$26,460
2015	$22,226.40