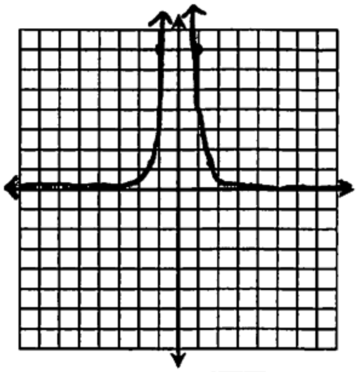
Unit 3: a little of everything
Thanksgiving Break
Homework

Topic 1: Graphing Power, Polynomial, and Rational Functions

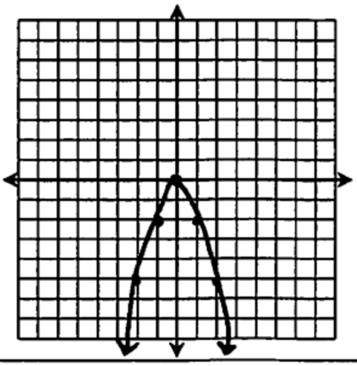
Graph each function and identify all key characteristics.

1. $f(x) = 7x^{-4}$



Domain: $\{x|x\neq 0\}$ **Range:** $\{y|y>0\}$
x-intercept(s): None
y-intercept: None
Inc. Interval(s): $(-\infty, 0)$
Dec. Interval(s): $(0, \infty)$
End Behavior:
As $x \to \infty$, $f(x) \to 0$
As $x \to -\infty$, $f(x) \to 0$

2. $f(x) = -2x^{\frac{4}{3}}$

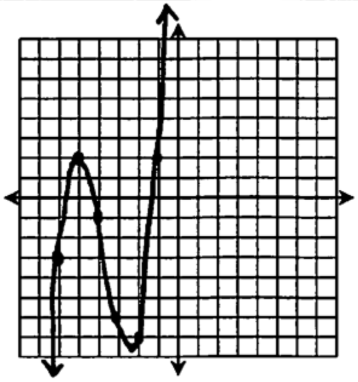


Domain: $\mathbb{R}$ **Range:** $\{y|y\leq 0\}$
x-intercept(s): $(0,0)$
y-intercept: $(0,0)$
Inc. Interval(s): $(-\infty, 0)$
Dec. Interval(s): $(0, \infty)$
End Behavior:
As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

3. $f(x) = x^3 + 11x^2 + 35x + 27$

Max $(-5,2)$

Min $(-2.\overline{3}, -7.48)$

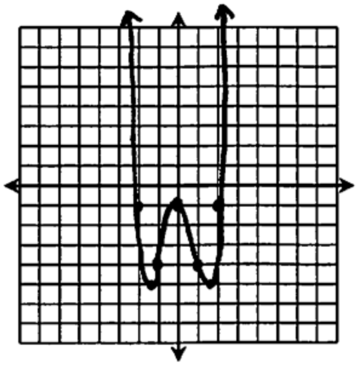


Domain: $\mathbb{R}$ **Range:** $\mathbb{R}$
x-intercept(s): $(-5.66,0)$, $(-4.2,0)$, $(-1.13,0)$
y-intercept: $(0,27)$
Inc. Interval(s): $(-\infty, -5)$, $(-2.\overline{3}, \infty)$
Dec. Interval(s): $(-5, -2.\overline{3})$
End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

4. $f(x) = x^4 - 4x^2 - 1$

Min $(-1.41, -5)$
$(1.41, -5)$

Max $(0,-1)$



Domain: $\mathbb{R}$ **Range:** $\{y|y\geq -5\}$
x-intercept(s): $(\pm 2.06, 0)$
y-intercept: $(0,-1)$
Inc. Interval(s): $(-1.41, 0)$, $(1.41, \infty)$
Dec. Interval(s): $(-\infty, -1.41)$, $(0, 1.41)$
End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

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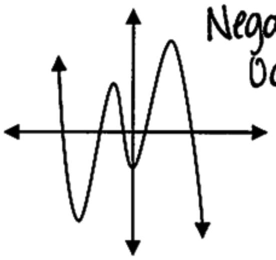
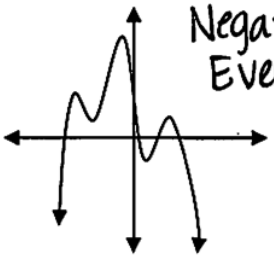
Given the graph of a polynomial functions below, determine the sign of the leading coefficient and whether the function has an even or odd degree.

9.



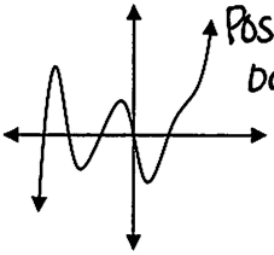
Negative, Odd

10.



Negative, Even

11.



Positive, odd

Topic 2: Dividing Polynomials

Divide the polynomials using synthetic or long division.

12. $(2x^4 - 8x^3 - 22x^2 - 15x + 20) \div (x - 6)$

$$\begin{array}{r|rrrrr} 6 & 2 & -8 & -22 & -15 & 20 \\ & \downarrow & 12 & 24 & 12 & -18 \\ \hline & 2 & 4 & 2 & -3 & 2 \end{array}$$

$$\boxed{2x^3 + 4x^2 + 2x - 3 + \frac{2}{x-6}}$$

13. $(5x^5 - 5x^4 + x^3 + 4x - 2) \div (x - 1)$

$$\begin{array}{r|rrrrrr} 1 & 5 & -5 & 1 & 0 & 4 & -2 \\ & \downarrow & 5 & 0 & 1 & 1 & 5 \\ \hline & 5 & 0 & 1 & 1 & 5 & -3 \end{array}$$

$$\boxed{5x^4 + x^2 + x + 5 - \frac{3}{x-1}}$$

14. $(20x^3 - 4x^2 + 10x + 1) \div (5x - 1)$

$$\begin{array}{r|l} & 4x^2 + 0x + 2 \\ \hline 5x-1 & 20x^3 - 4x^2 + 10x + 1 \\ & -(20x^3 - 4x^2) \\ \hline & 0x^2 + 10x \\ & -(0x^2 + 0x) \\ \hline & 10x + 1 \\ & -(10x - 2) \\ \hline & 3 \end{array}$$

$$\boxed{4x^2 + 2 + \frac{3}{5x-1}}$$

15. $(3x^3 + 20x^2 + 14x - 8) \div (x^2 + 6x + 1)$

$$\begin{array}{r|l} & 3x + 2 \\ \hline x^2 + 6x + 1 & 3x^3 + 20x^2 + 14x - 8 \\ & -(3x^3 + 18x^2 + 3x) \\ \hline & 2x^2 + 11x - 8 \\ & -(2x^2 + 12x + 2) \\ \hline & -x - 10 \end{array}$$

$$\boxed{3x + 2 + \frac{-x-10}{x^2+6x+1}}$$

Topic 3: The Remainder Theorem

Use the Remainder Theorem to evaluate $f(x)$ at $x = c$.

16. $f(x) = x^3 - 4x^2 + x + 6$; $c = 5$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & 1 & 6 \\ & \downarrow & 5 & 5 & 30 \\ \hline & 1 & 1 & 6 & 36 \end{array}$$

$\boxed{36}$

17. $f(x) = 4x^4 - 3x^3 + 7x - 11$; $c = -2$

$$\begin{array}{r|rrrrr} -2 & 4 & -3 & 0 & 7 & -11 \\ & \downarrow & -8 & 22 & -44 & 74 \\ \hline & 4 & -11 & 22 & -37 & 63 \end{array}$$

$\boxed{63}$

18. $f(x) = x^4 - 6x^3 - 3x^2 + 2x + 2$; $c = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & -3 & 2 & 2 \\ & \downarrow & 1 & -5 & -8 & -6 \\ \hline & 1 & -5 & -8 & -6 & -4 \end{array}$$

$\boxed{-4}$

19. $f(x) = 2x^4 + 5x^3 + 3x + 4$; $c = -3$

$$\begin{array}{r|rrrrr} -3 & 2 & 5 & 0 & 3 & 4 \\ & \downarrow & -6 & 3 & -9 & 18 \\ \hline & 2 & -1 & 3 & -6 & 22 \end{array}$$

$\boxed{22}$

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Topic 4: The Factor Theorem

Use the factor theorem to determin which binomials are factors of the functions below.

20. $f(x) = 2x^3 - 3x^2 - 17x - 12$

(X+1):

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -17 & -12 \\ & \downarrow & -2 & 5 & 12 \\ \hline & 2 & -5 & -12 & \textcircled{0} \end{array}$$

(X−4):

$$\begin{array}{r|rrr} 4 & 2 & -5 & -12 \\ & \downarrow & 8 & 12 \\ \hline & 2 & 3 & \textcircled{0} \end{array}$$

(X+3):

$$\begin{array}{r|rr} -3 & 2 & 3 \\ & \downarrow & -6 \\ \hline & 2 & \cancel{-3} \end{array}$$

☑ $(x+1)$
☑ $(x-4)$
☐ $(x+3)$

21. $f(x) = x^4 + 4x^3 - 8x^2 - 35x - 12$

(X+2):

$$\begin{array}{r|rrrrr} -2 & 1 & 4 & -8 & -35 & -12 \\ & \downarrow & -2 & -4 & 24 & 22 \\ \hline & 1 & 2 & -12 & -11 & \cancel{10} \end{array}$$

(X+4):

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & -8 & -35 & -12 \\ & \downarrow & -4 & 0 & 32 & 12 \\ \hline & 1 & 0 & -8 & -3 & \textcircled{0} \end{array}$$

(X−6):

$$\begin{array}{r|rrrr} 6 & 1 & 0 & -8 & -3 \\ & \downarrow & 6 & 36 & 168 \\ \hline & 1 & 6 & 28 & \cancel{165} \end{array}$$

☐ $(x+2)$
☑ $(x+4)$
☐ $(x-6)$

Topic 5: Rational Zero Theorem

Use the Rational Zero Theorem to list all possible rational zeros.

22. $f(x) = x^4 + 12x^3 + 7x^2 - 42$

±1, ±2, ±3, ±6, ±7, ±14, ±21, ±42

23. $f(x) = x^3 - x^2 - 8x^2 + 15$

±1, ±3, ±5, ±15

24. $f(x) = 3x^5 - 11x^3 - 15x + 24$

±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24,
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

25. $f(x) = 2x^3 + 5x^2 - 2x + 28$

±1, ±2, ±4, ±7, ±14, ±28,
$\pm\frac{1}{2}, \pm\frac{7}{2}$

Topic 6: Descartes' Rule of Signs

Use Descartes' Rule of Signs to give the possible number of positive and negative real zeros.

26. $f(x) = -x^5 + 18x^3 + 6x^2 - 5x - 4$

$f(-x) = x^5 - 18x + 6x^2 + 5x - 4$

Pos: 2 or 0   Neg: 3 or 1

27. $f(x) = x^4 + 7x^3 - 9x^2 + x - 2$

$f(-x) = x^4 - 7x^3 - 9x^2 - x - 2$

Pos: 3 or 1   Neg: 1

28. $f(x) = 5x^4 - 14x^2 + 9$

$f(-x) = 5x^4 - 14x^2 + 9$

Pos: 2 or 0   Neg: 2 or 0

29. $f(x) = 6x^5 - 3x^4 + 56x^3 - 28x^2 + 64x - 32$

$f(-x) = -6x^5 - 3x^4 - 5x^3 - 28x^2 - 64x - 32$

Pos: 5, 3, or 1   Neg: 0

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Topic 7: Zeros of Polynomial Functions & Complete Factorization

Find all zeros and give the complete factorization of the function. Use the Rational Zero Theorem and division when necessary. Simplify all irrational and complex solutions.

30. $f(x) = x^4 + 14x^2 - 72$

$f(x) = (x^2-4)(x^2+18)$

$x = \pm 2$ | $x^2 = -18$

$x = \pm i\sqrt{18}$

$x = \pm 3i\sqrt{2}$

Zeros: $x = \{\pm 2, \pm 3i\sqrt{2}\}$

$\boxed{f(x) = (x+2)(x-2)(x+3i\sqrt{2})(x-3i\sqrt{2})}$

31. $f(x) = 5x^3 + 12x^2 + x - 6$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/5, \pm 2/5, \pm 3/5, \pm 6/5$

$$\begin{array}{r|rrrr} -1 & 5 & 12 & 1 & -6 \\ & \downarrow & -5 & -7 & 6 \\ \hline & 5 & 7 & -6 & 0 \end{array}$$

$f(x) = (x+1)(5x^2+7x-6)$

$\boxed{f(x) = (x+1)(5x-3)(x+2)}$

zeros: $\boxed{x = \{-2, -1, 3/5\}}$

32. $f(x) = x^3 - 3x^2 - 13x + 15$

$\pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -13 & 15 \\ & \downarrow & 1 & -2 & -15 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$f(x) = (x-1)(x^2-2x-15)$

$\boxed{f(x) = (x-1)(x-5)(x+3)}$

zeros: $\boxed{x = \{-3, 1, 5\}}$

33. $f(x) = x^3 + 2x^2 - 17x - 36$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

$$\begin{array}{r|rrrr} -4 & 1 & 2 & -17 & -36 \\ & \downarrow & -4 & 8 & 36 \\ \hline & 1 & -2 & -9 & 0 \end{array}$$

$f(x) = (x+4)(x^2-2x-9)$

$x = -4$ | $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{40}}{2}$

$x = \dfrac{2 \pm 2\sqrt{10}}{2}$

$x = 1 \pm \sqrt{10}$

$\boxed{f(x) = (x+4)(x-(1+\sqrt{10}))(x-(1-\sqrt{10}))}$

zeros: $\boxed{x = \{-4, 1 \pm \sqrt{10}\}}$

Topic 8: Multiplicity

Identify the zeros, their multiplicities, and describe the effect on the graph.

34. $f(x) = x^3(2x-3)^2(x+7)^5$

Zero	Multiplicity	Effect
-7	5	intersects
0	3	intersects
3/2	2	tangent

35. $f(x) = x^6 - 2x^5 - 4x^4 + 8x^3$

$x^5(x-2) - 4x^3(x-2)$

$(x^5 - 4x^3)(x-2)$

$x^3(x^2-4)(x-2)$

$x^3(x+2)(x-2)^2$

Zero	Multiplicity	Effect
-2	1	intersects
0	3	intersects
2	2	tangent

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Topic 9: Writing Polynomial Functions Given Zeros

Use the given zeros to write a polynomial function.

36. 0, −1 (multiplicity 2), $\frac{4}{3}$

$x(x+1)^2(3x-4)$

$=(3x^2-4x)(x^2+2x+1)$

$=3x^4+6x^3+3x^2-4x^3-8x^2-4x$

$\boxed{f(x)=3x^4+2x^3-5x^2-4x}$

37. ±5, $\pm 3\sqrt{2}$

$(x+5)(x-5)(x+3\sqrt{2})(x-3\sqrt{2})$

$=(x^2-25)(x^2-18)$

$=x^4-18x^2-25x^2+450$

$\boxed{f(x)=x^4-43x^2+450}$

38. −3, −1−3*i*

$(x+3)(x-(-1-3i))(x-(-1+3i))$

$=(x+3)\left(x^2-x(-1+3i)-x(-1-3i)+(-1-3i)(-1+3i)\right)$

$=(x+3)\left(x^2+x-3ix+x+3ix+10\right)$

$=(x+3)(x^2+2x+10)$

$=x^3+2x^2+10x+3x^2+6x+30$

$\boxed{f(x)=x^3+5x^2+16x+30}$

Topic 10: Polynomial & Rational Inequalities

Solve the inequality. Use the number line to test intervals.

39. $2x^3+5x^2-32x-80>0$

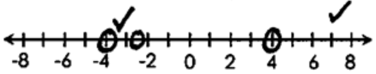
$(x^2-16)(2x+5)>0$ Zeros: $-4, -\frac{5}{2}, 4$

$-5: 2(-5)^3+5(-5)^2-32(-5)-80>0;\ -45>0$ ✗

$-3: 2(-3)^3+5(-3)^2-32(-3)-80>0;\ 7>0$ ✓

$0: 2(0)^3+5(0)^2-32(0)-80>0;\ -80>0$ ✗

$5: 2(5)^3+5(5)^2-32(5)-80>0;\ 135>0$ ✓



$\boxed{\left(-4,-\frac{5}{2}\right)\cup(4,\infty)}$

40. $x^3-37x+45\le 4x^2+5$

$x^3-4x^2-37x+40\le 0$

$\pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm20, \pm40$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -37 & 40 \\ & \downarrow & 1 & -3 & -40 \\ \hline & 1 & -3 & -40 & 0 \end{array}$$

$f(x)=(x-1)(x^2-3x-40)$

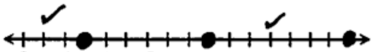
$(x-1)(x-8)(x+5)$

Zeros: $-5, 1, 8$

$-6: (-6)^3-4(-6)^2-37(-6)+40\le 0;\ -98\le 0$ ✓

$0: 0^3-4(0)^2-37(0)+40\le 0;\ 40\le 0$ ✗

$2: 2^3-4(2)^2-37(2)+40\le 0;\ -42\le 0$ ✓



$9: 9^3-4(9)^2-37(9)+40\le 0;$
$112\le 0$ ✗

$\boxed{(-\infty,-5]\cup[1,8]}$

41. $\frac{x^2+x-12}{x-1}>0$

Zeros: $x=-4, 3$

Asym: $x=1$

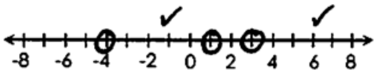
$\frac{(x+4)(x-3)}{x-1}>0$

$-5: \frac{8}{-6}>0;\ -\frac{4}{3}>0$ ✗

$0: \frac{-12}{-1}>0;\ 12>0$ ✓

$2: \frac{-6}{1}>0;\ -6>0$ ✗

$4: \frac{8}{3}>0$ ✓



$\boxed{(-4,1)\cup(3,\infty)}$

42. $\frac{7}{x+3}\le\frac{6}{x+2}$

$\frac{7(x+2)-6(x+3)}{(x+3)(x+2)}\le 0$

Zero: $x=4$

Asym: $x=-3, -2$

$\frac{x-4}{(x+3)(x+2)}\le 0$

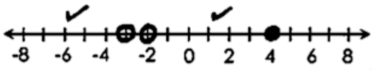
$-4: \frac{-8}{2}\le 0;\ -4\le 0$ ✓

$-2.5: \frac{-6.5}{-.25}\le 0;$

$26\le 0$ ✗

$0: \frac{-4}{6}\le 0;\ \frac{-2}{3}\le 0$ ✓

$5: \frac{1}{56}\le 0$ ✗



$\boxed{(-\infty,-3)\cup(-2,4]}$

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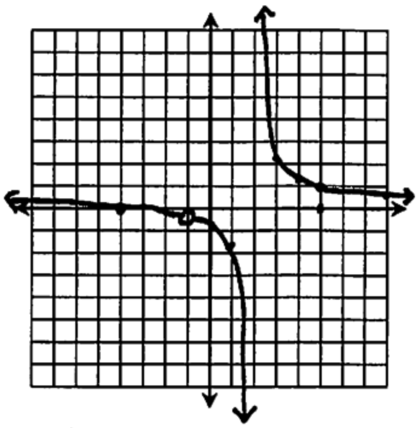
5. $f(x)=\dfrac{x^2+5x+4}{3x^2-3x-6}$

$= \dfrac{(x+4)(x+1)}{3(x-2)(x+1)}$

$= \dfrac{x+4}{3(x-2)}$

Hole: $x=-1$



Domain: $\{x \mid x \neq -1, 2\}$ **Range:** $\{y \mid y \neq -\frac{1}{3}, \frac{1}{3}\}$

***x*-intercept(s):** $(-4, 0)$

***y*-intercept:** $(0, -2/3)$

Inc. Interval(s): None

Dec. Interval(s): $(-\infty, 2), (2, \infty)$

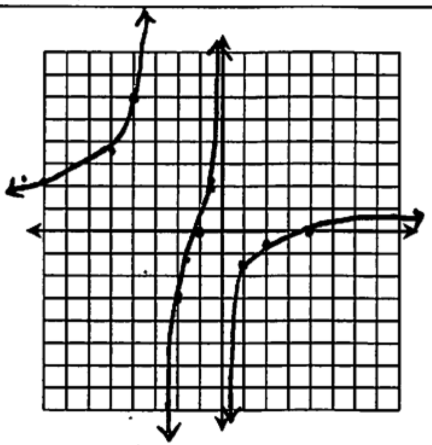
VA: $x=2$ **HA:** $y = 1/3$

SA: None

Holes: $(-1, -1/3)$

6. $f(x)=\dfrac{x^2-3x-4}{x^2+3x}$

$= \dfrac{(x-4)(x+1)}{x(x+3)}$



Domain: $\{x \mid x \neq -3, 0\}$ **Range:** $\{y \mid y \neq 1\}$

***x*-intercept(s):** $(-1, 0), (4, 0)$

***y*-intercept:** None

Inc. Interval(s): $(-\infty, -3), (-3, 0), (0, \infty)$

Dec. Interval(s): None

VA: $x=-3, x=0$ **HA:** $y=1$

SA: None

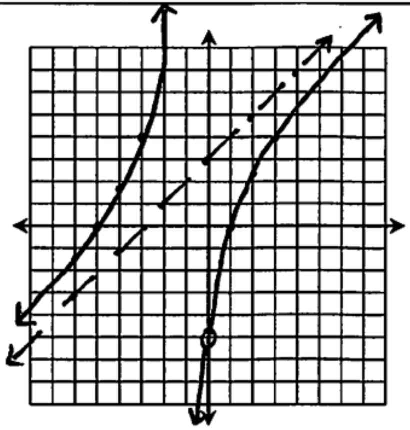
Holes: None

7. $f(x)=\dfrac{2x^3+8x^2-10x}{2x^2+2x}$

$= \dfrac{2x(x+5)(x-1)}{2x(x+1)}$

$= \dfrac{x^2+4x-5}{x+1}$

$$\begin{array}{r|rrr} -1 & 1 & 4 & -5 \\ & \downarrow & -1 & -3 \\ \hline & 1 & 3 & -8 \end{array}$$



Domain: $\{x \mid x \neq -1, 0\}$ **Range:** $\mathbb{R}$

***x*-intercept(s):** $(-5, 0), (1, 0)$

***y*-intercept:** None

Inc. Interval(s): $(-\infty, -1), (-1, \infty)$

Dec. Interval(s): None

VA: $x=-1$ **HA:** None

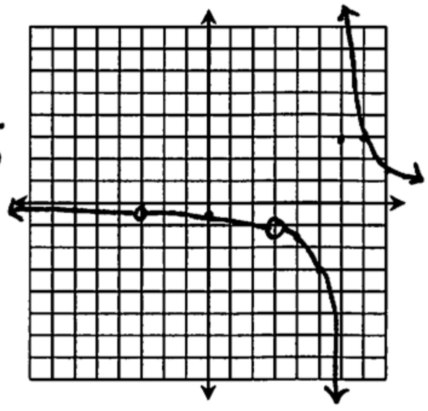
SA: $y=x+3$

Holes: $(0, -5)$

8. $f(x)=\dfrac{3x^2-27}{x^3-6x^2-9x+54}$

$= \dfrac{3(x+3)(x-3)}{(x-6)(x+3)(x-3)}$

$= \dfrac{3}{x-6}$



Domain: $\{x \mid x \neq \pm 3, 6\}$ **Range:** $\{y \mid y \neq -1, -\frac{1}{3}, 0\}$

***x*-intercept(s):** None

***y*-intercept:** $(0, -0.5)$

Inc. Interval(s): None

Dec. Interval(s): $(-\infty, 6), (6, \infty)$

VA: $x=6$ **HA:** $y=0$

SA: None

Holes: $(-3, -1/3)(3, -1)$

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Unit 4

Exponentials and Logarithmic Functions

CONCEPT

Application of Growth and Decay

The exponential growth or decay can be modeled by $P_t = P_0 e^{rt}$

where P_0 is the initial value
P_t is the value at time t
r is a growth/decay rate
($r > 0$ for growth and $r < 0$ for decay)
t is the time

The common applications of exponential growth and decay are population (of a town or bacteria) or decay of radioactive substance.

Usually, you are provided with all the information except the one variable you are solving for.

The initial population of a bacteria in a culture is 300. If the growth is exponential and the rate of growth is 23% for every hour, what will be the population after 6 hours?

The initial population is 300 so $P_0 = 300$
The rate of growth is 23% so $r = 0.23$
The time is 6 hours so $t = 6$ is defined hourly.

To find the population after 6 hours, substitute the values into $P_t = P_0 e^{rt}$.

$$P_6 = 300e^{0.23 \cdot 6} = 300e^{1.38} \approx 1192$$

So the population after 6 hours is 1192.

The following pages are copies of your notes/homework. They should be in the order they were given to you.

There are copies of ONLY the homework at the end as well.

Unit 4
Exponential Functions
& Natural Bases

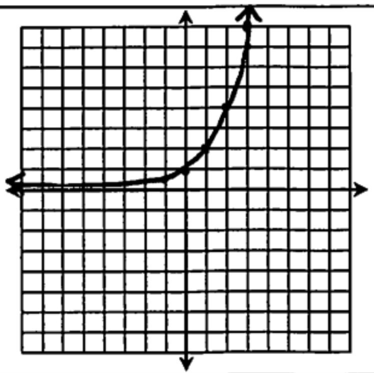
Main Ideas/Questions | **Notes/Examples**

EXPONENTIAL FUNCTION

$f(x)=ab^x$

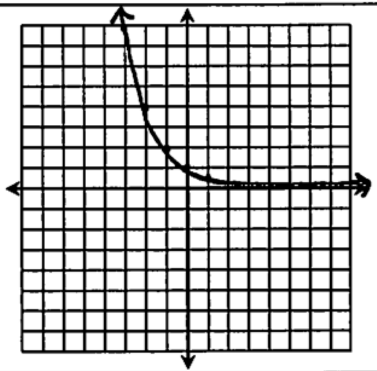
b is the **base** of the function

Graph $f(x)=2^x$



When $a>1$ and $b>1$, the function is **increasing** and called an **exponential growth.**

Graph $f(x)=\left(\frac{1}{2}\right)^x$



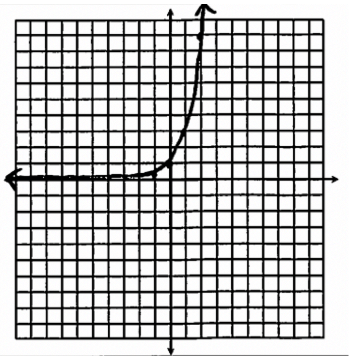
When $a<1$ and $b<1$, the function **decreasing** and called an **exponential decay.**

Directions: Classify the function as an exponential growth or decay, graph,

1. $f(x)=3^x$

Growth

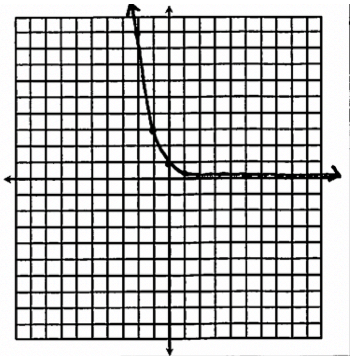
Increasing interval? - ∞ → ∞



2. $f(x)=\left(\frac{1}{3}\right)^x$

Decay

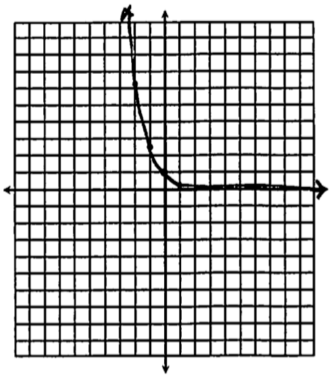
Decreasing interval? - ∞ → ∞



3. $f(x)=\left(\frac{2}{5}\right)^x$

Decay

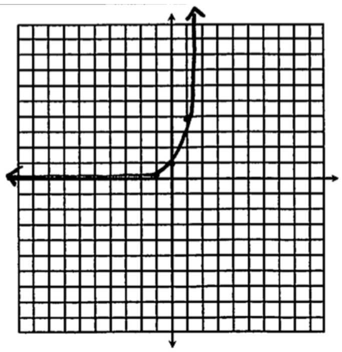
Decreasing interval? - ∞ → ∞



4. $f(x)=4^x$

Growth

Increasing interval? - ∞ → ∞



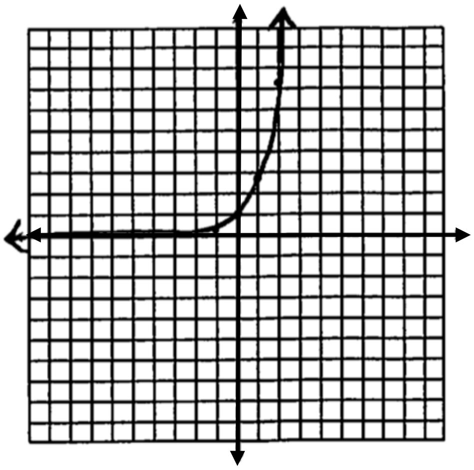
Natural Base

EXPONENTIAL FUNCTION

$f(x) = e^x$

- *e* is an irrational number with an approximate value of 2.71828.
- Exponential functions with base *e* are called **natural base** exponential functions.
- Many real-world applications of exponential functions use base *e*.

Graph the function $f(x) = e^x$, then identify its key characteristics.



Growth or Decay? growth	
Domain: ℝ	**Range:** {y | y>0}
y-intercept: (0,1)	**Asymptote:** y=0
Increasing Interval: $-\infty \rightarrow \infty$	
Decreasing Interval: none	
End Behavior: As $x \rightarrow \infty$, $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$, $f(x) \rightarrow 0$	

Transformations of

EXPONENTIAL FUNCTIONS

Recall the following transformations rules given a function $f(x)$:

Translations (Shifts)	**Reflections**	**Dilations** (compress/stretch)
$f(x+h)$ shifts left	$-f(x)$ reflects over the x-axis	$a \cdot f(x)$ is a vertical compression when $\lvert a \rvert < 1$ and a vertical stretch when $\lvert a \rvert > 1$
$f(x+h)$ shifts right		
$f(x)+k$ shifts up	$f(-x)$ reflects over the y-axis	$f(b \cdot x)$ is a horizontal stretch when $\lvert b \rvert < 1$ and a horizontal compression when $\lvert b \rvert > 1$
$f(x)-k$ shifts down		

Directions: (a) Identify the parent function, and **(b)** describe the transformations.

1. $f(x) = 2^{x+1} - 3$

a) $f(x) = 2^x$ $\quad f^{(x+1)} -3$

b) 2^{x+1}: translates left 1
-3: down 3

2. $f(x) = -\left(\frac{1}{2}\right)^{x-5} + 1$

a) $f(x) = \left(\frac{1}{2}\right)^x$

b) $-f(x)$: reflected over x-axis
x-5: right 5 +1: up 1

3. $f(x) = -3 \cdot 4^{x-2} - 7$

a) $f(x) = 4^x$ $\quad -3\ f^{(x-2)} -7$

b) -3: reflect over x-axis & vert. stretch by 3
(x-2): right 2
-7: down 7

4. $f(x) = \frac{4}{3} \cdot e^{-x} + 5$

a) $f(x) = e^x$

b) Vertical stretch by $\frac{4}{3}$;
Reflect across the y-axis;
Translate up 5

-- Unit 4 --
Exponential Growth and Decay
Homework

Directions: Classify each function as an exponential growth or an exponential decay. Sketch the curve.

1. $f(x)=\frac{1}{7}\cdot 6^x$

2. $f(x)=\frac{3}{2}\cdot\left(\frac{1}{4}\right)^x$

3. $f(x)=3\cdot\left(\frac{5}{2}\right)^x$

Directions: (a) Identify the parent function and **(b)** describe the transformations.

4. $f(x)=-\left(\frac{4}{3}\right)^{2(x-3)}+1$

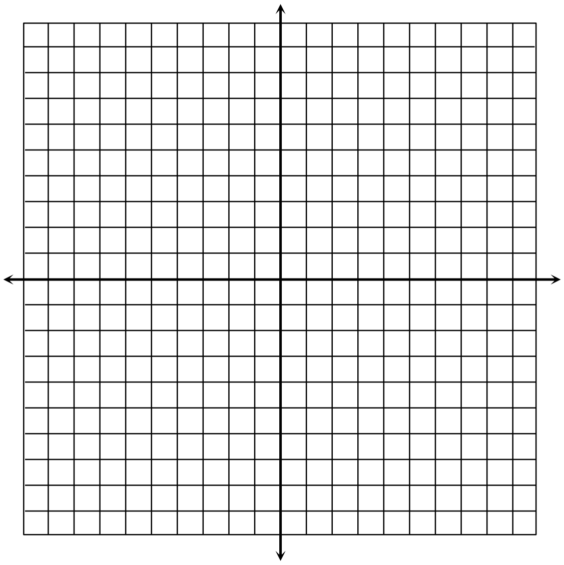
5. $f(x)=7^{-\frac{3}{2}x}+3$

6. $f(x)=\frac{1}{3}\cdot e^{-x}-9$

7. $f(x)=5\cdot\left(\frac{4}{5}\right)^{x+3}$

Directions: Graph each function, then identify its key characteristics.

8. $f(x)=\frac{1}{2}\cdot 5^{-x}-1$



Domain:

Range:

y-intercept:

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

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-- Unit 4 --
Growth and Decay

(population, $$$, bacteria, etc.)

Main Ideas/Questions

Exponential GROWTH & DECAY

a = initial amount

r = rate (decimal)

t = length of time

GROWTH	DECAY
Exponential growth occurs when a quantity **exponentially increases** over time.	Exponential decay occurs when a quantity **exponentially decreases** over time.
EXPONENTIAL GROWTH FUNCTION: $f(t) = a(1+r)^t$	**EXPONENTIAL DECAY FUNCTION:** $f(t) = a(1-r)^t$

where ***a*** is the initial amount, ***r*** is the growth or decay rate (as a decimal), and ***t*** is the length of time

EXAMPLES:

If calculated monthly, your "t" will be the # of months.

If a half life is 3 days, then "t" value will be t/3 (every 3 days, "t" will change)

1. Brooke started her career with an annual salary of $32,000. Each year thereafter, her salary increased by 2.5%. Write and use an exponential growth function to find her salary when she retires after 30 years.

$f(t) = a(1+r)^t$

$f(t) = 32000\ (1+\ .025)^t$

$f(30) = 32,000\ (1.025)^{30}$

$= \$67,122.16$

2. In 1995, a magazine had 14,000 subscribers. The number of subscribers increased by 40% each year thereafter. Write and use an exponential growth function to find the number of subscribers in 2016.

$f(t) = a(1+r)^t$

$f(t) = 14,000\ (1+.40)^t$

t is difference in years

$f(21) = 14,000\ (1.40)^{21}$

$= 16,398,978$

3. Kate drank an energy beverage with 150 milligrams of caffeine. Each hour the amount of caffeine in her system decreases by about 12%. Write and use an exponential decay function to find the amount of caffeine in her system after eight hours.

$f(t) = a(1-r)^t$

$f(t) = 150\ (1-.12)^t$

$f(8) = 150\ (0.88)^8$

$= 53.95$ mL

4. The half-life of Mercury-197 is 3 days. Write and use an exponential decay function to find the amount of Mercury-197 left from a 50-gram sample after 20 days.

$f(t) = a(1-r)^t$

$f(t) = 50\ (1-.5)^{t/3}$

$f(20) = 50\ (0.5)^{20/3}$

$= 0.49$ grams

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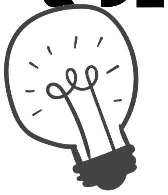
Continuous

GROWTH & DECAY



$e = 2.71828$

Sometimes a quantity is constantly increasing or decreasing at an exponential rate, and not just after each year, month, day, hour, etc. The formula to the right can be used to find the balance of the account in this case.

$A=Pe^{rt}$

+r = growth

-r = decay

for decay models

5. A garbage dumpster started with 4 pounds of garbage. The amount of garbage increased continuously by 35% each day from this point forward. Find the amount of garbage in the dumpster after two weeks.

$A = 4e^{0.35(14)}$

$A = 4e^{4.9}$

$A = 537.16$ lb

6. The population of a town is declining at a continuous rate of 1.5%. If the current population is 16,000 people, find the population in 8 years.

$A = 16000\, e^{-.015(8)}$

$A = 16000\, e^{-.12}$

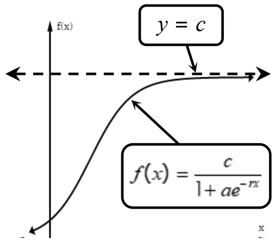
$\boxed{A = 14190 \text{ people}}$

LOGISTIC GROWTH

Function

Sometimes a quantity exponential increases, but then levels out, approaching a horizontal asymptote. This is called a **logistic growth model**. The logistic growth function is given as:

$$f(x) = \frac{c}{1+ae^{-rx}}$$



7. A disease begins to spread in a town of 20,000 people. After *t* days, the number of people who have been infected by the disease is modeled by the function below. Using the function, find the number of people infected after 10 days.

$$f(t) = \frac{20{,}000}{1+1150e^{-0.95t}}$$

$$f(10) = \frac{20000}{1+1150e^{-.95(10)}}$$

$\approx \boxed{18415 \text{ people}}$

8. The population *P*, in millions, of a country from 1850 to 2000 is modeled by the equation below where *t* is the years since 1850. Using the function, find the population of the country in 1920.

$$P(t) = \frac{135}{1+58e^{-0.025t}}$$

$$P(70) = \frac{135}{1+58\, e^{-.025(70)}}$$

$= \boxed{12.19 \text{ million}}$

-- Unit 4 --
Growth and Decay

Homework

Exponential Growth and Decay

1. Aaron owns a rare baseball card. He bought the card for $7.50 in 1987 and its value increases by 6% each year. Write and use an exponential growth function to find the baseball card's value in 2015.

2. Jennifer started working at her job earning $6.25 per hour. Every six months, she gets a 3.25% raise. If Jennifer has worked at the job for 14 years, what is her hourly rate?

3. In 2005, the Summerville Journal had 110,000 subscriptions. The number of subscriptions subsequently decreased by 8% each year. Write and use an exponential decay function to find the number of subscriptions in 2022.

4. In November, 26 students at Monarch High School had contracted the flu. Each month, the number of students who have contracted the flu increases by 36%. Write and use an exponential growth function to find the total number of students who have contracted the flu by May.

5. Ian bought a new truck for $35,000 in 2015. Each year, the value of the truck depreciates by 9%. Write and use an exponential growth function to find the value of his truck at the end of his 60-month loan.

6. A certain compound has a half-life of four days. Write and use an exponential decay function to find the amount of compound remaining from a 75-ounce sample after three weeks.

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Continuous Growth and Decay

7. A 4-foot tree was planted in 1984. The tree grows continuously by 22% each year from this point forward. Find the height of the tree after 8 years.

8. An ice sculpture measures 52 inches and melts continuously by 3% per minute. Find the height of a sculpture after 15 minutes.

9. In 2002, a certain town recorded 15,300 acres of undeveloped land. Each year after, the amount of land decreased by 7% due to residential and commercial development. Find the approximate amount of undeveloped land in 2013.

Logistic Growth

10. The population of fish in a pond from 2001 to 2014 is modeled by the function below, where t is the years since 2001. Using the function, find the number of fish in the pond in 2014.

$$P(t)=\frac{1125}{1+12e^{-0.17t}}$$

11. The bears in Alaska are limited to a certain area to live due to the resources available for food and shelter. After t years, the number of bears living in the area is modeled by the function below. Using the function, find the number of bears after 17 years.

$$f(t)=\frac{103}{1+26e^{-0.31t}}$$

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-- Unit 4 --
Compound Interest
$$$$$$$$$

Main Ideas/Questions | **Notes/Examples**

COMPOUND INTEREST

A common application of exponential growth is compound interest. Compound interest is interest paid on both the initial investment, called the principal, and on previously earned interest.

FORMULA:

$$A = P(1+\frac{r}{n})^{nt}$$

A = total balance

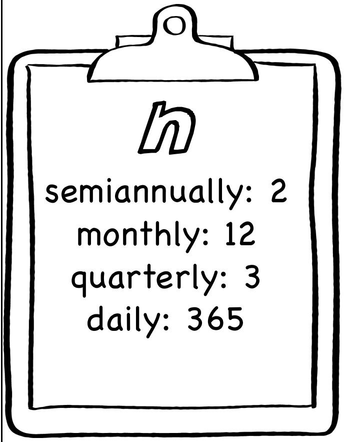
P = principal (initial) amount

r = rate

n = # times compounded (yearly)

t = time

EXAMPLES



n

semiannually: 2

monthly: 12

quarterly: 3

daily: 365

1. Dave invests $300 in an account with a 5% interest rate. If he makes no other deposits or withdrawals, find his account balance after 15 years if the interest is compounded with the following frequencies.

a) semiannually

$$A = 300(1+\frac{.05}{2})^{2(15)}$$

$$A = 300(1.025)^{30}$$

$$A = \$629.27$$

b) monthly

$$A = 300(1+\frac{.05}{12})^{12(15)}$$

$$A = 300(1.00416)^{180}$$

$$A = \$634.11$$

2. If $2,500 is deposited into a savings account earning 8% annual interest, how much will be in the account at the end of 25 years if the interest is compounded with the following frequencies:

a) quarterly (n=4)

$$A = 2500\left(1+\frac{.08}{4}\right)^{4(25)}$$

$$A = 2500(1.02)^{100}$$

$$\boxed{A = \$18111.62}$$

b) daily (n=365)

$$A = 2500\left(1+\frac{.08}{365}\right)^{365(25)}$$

$$A = 2500(1.000219)^{9125}$$

$$\boxed{A = \$18468.58}$$

3. When Amelia turned 6, her grandparents opened a college savings account for her with an initial deposit of $500. The account earns 3.2% interest compounded bimonthly. If her grandparents make no other deposits or withdrawals, how much money will be in the account when Amelia can access it at age 18? (t=12, n=24) n= (2)12

$$A = 500\left(1+\frac{.032}{24}\right)^{24(12)}$$

$$A = 500(1.001\overline{3})^{288}$$

$$\boxed{A = \$733.88}$$

-- Unit 4 --
Compound Interest
$$$$$$$$$

Main Ideas/Questions	Notes/Examples
COMPOUND INTEREST	A common application of exponential growth is compound interest. Compound interest is interest paid on both the initial investment, called the principal, and on previously earned interest.
	FORMULA: $A = P\left(1+\frac{r}{n}\right)^{nt}$ A = total balance P = principal (initial) amount r = rate n = # times compounded (yearly) t = time
EXAMPLES	**1.** Dave invests $300 in an account with a 5% interest rate. If he makes no other deposits or withdrawals, find his account balance after 15 years if the interest is compounded with the following frequencies.
	a) semiannually (n=2) $A = 300\left(1+\frac{.05}{2}\right)^{2(15)}$ $A = 300(1.025)^{30}$ $\boxed{A = \$629.27}$
	b) monthly (n=12) $A = 300\left(1+\frac{.05}{12}\right)^{12(15)}$ $A = 300(1.0041\overline{6})^{180}$ $\boxed{A = \$634.11}$
	2. If $2,500 is deposited into a savings account earning 8% annual interest, how much will be in the account at the end of 25 years if the interest is compounded with the following frequencies:
	a) quarterly (n=4) $A = 2500\left(1+\frac{.08}{4}\right)^{4(25)}$ $A = 2500(1.02)^{100}$ $\boxed{A = \$18111.62}$
	b) daily (n=365) $A = 2500\left(1+\frac{.08}{365}\right)^{365(25)}$ $A = 2500(1.000219)^{9125}$ $\boxed{A = \$18468.58}$
	3. When Amelia turned 6, her grandparents opened a college savings account for her with an initial deposit of $500. The account earns 3.2% interest compounded bimonthly. If her grandparents make no other deposits or withdrawals, how much money will be in the account when Amelia can access it at age 18? (t=12, n=24) $A = 500\left(1+\frac{.032}{24}\right)^{24(12)}$ $\boxed{A = \$733.88}$ $A = 500(1.001\overline{3})^{288}$

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4. Suppose a savings account offers a 0.4% interest rate compounded semiannually. If Samantha opens an account with \$750 and makes no other deposits or withdrawals, how much interest will she have earned after 10 years?

5. In 1990, Carter deposited \$1,000 in an investment account that earns $2\frac{3}{8}\%$ annual interest, compounded quarterly. If no other deposits or withdrawals were made, find the balance of his account in 2025.

CONTINUOUS COMPOUND INTEREST

In some cases, interest is compounded **continuously** meaning the account is constantly earning interest. The formula to the right can be used to find the balance of the account in this case.

FORMULA:

$$A=Pe^{rt}$$

EXAMPLES

6. Suppose \$800 is invested in an account at a 6% interest rate compounded continuously. If no other withdrawals or deposits are made, find the balance in the account after 20 years.

$$A=800e^{.06(20)}$$

$$A=800e^{1.2}$$

$$\boxed{A= \$2656.09}$$

7. Find the balance of an account after 5 years if \$1,200 is initially invested at an interest rate of 12.5% per year, compounded continuously and there are no other deposits or withdrawals.

Option A:
5.5% annual interest
compounded
monthly

Option B:
2.7% annual interest
compounded
continuously

8. Carla is investing \$1,500 in a new 30-year retirement account. Determine which of the interest rates and compounding periods shown to the left would be her best investment option.

4. Suppose a savings account offers a 0.4% interest rate compounded semiannually. If Samantha opens an account with $750 and makes no other deposits or withdrawals, how much interest will she have earned after 10 years?

$A = 750\left(1 + \frac{.004}{2}\right)^{10(2)}$

$A = 750(1.002)^{20}$

$A = 780.58$

$\boxed{\$30.58 \text{ in interest}}$

5. In 1990, Carter deposited $1,000 in an investment account that earns $2\frac{3}{8}$% annual interest, compounded quarterly. If no other deposits or withdrawals were made, find the balance of his account in 2025.

$A = 1000\left(1 + \frac{.02375}{4}\right)^{4(35)}$

$A = 1000(1.0059375)^{140}$

$\boxed{A = \$2290.55}$

CONTINUOUS COMPOUND INTEREST

In some cases, interest is compounded **continuously** meaning the account is constantly earning interest. The formula to the right can be used to find the balance of the account in this case.

FORMULA:

$A = Pe^{rt}$

EXAMPLES

6. Suppose $800 is invested in an account at a 6% interest rate compounded continuously. If no other withdrawals or deposits are made, find the balance in the account after 20 years.

$A = 800e^{.06(20)}$

$A = 800e^{1.2}$

$\boxed{A = \$2656.09}$

7. Find the balance of an account after 5 years if $1,200 is initially invested at an interest rate of 12.5% per year, compounded continuously and there are no other deposits or withdrawals.

$A = 1200e^{.125(5)}$

$A = 1200e^{.625}$

$\boxed{A = \$2241.90}$

Option A:
5.5% annual interest compounded monthly

Option B:
2.7% annual interest compounded continuously

8. Carla is investing $1,500 in a new 30-year retirement account. Determine which of the interest rates and compounding periods shown to the left would be her best investment option.

$A = 1500\left(1 + \frac{.055}{12}\right)^{12(30)}$

$A = \$7781.08$

$A = 1500e^{.027(30)}$

$A = \$3371.86$

$\boxed{\text{Option A yields more money.}}$

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HW

Compound Interest

1. If $1,800 is deposited into an account earning 6% interest, how much will be in the account at the end of 18 years if the interest is compounded with the following frequencies:

a) quarterly

b) weekly

2. Erica was given $300 for her birthday and decided to put it in a savings account that earns 3.75% interest. If she makes no other deposits or withdrawals, find her account balance after ten years if the interest is compounded with the following frequencies.

a) semiannually

b) daily

3. A $2,750 deposit was made to an account earning $2\frac{3}{4}\%$ annual interest compounded weekly. If no other deposits or withdrawals are made, find the balance of the account after nine years.

Continuous Compound Interest

7. Moises was given a $1,500 signing bonus at his new job. He is going to invest this money in an account that earns 6% interest, compounded continuously. Find the account balance after ten years.

8. Suppose $2,800 is deposited into an account at a 2.5% interest rate, compounded continuously. If there are no other deposits or withdrawals, find the account balance after 25 years.

9. Find the balance of an account after seven years if $600 is deposited and the interest rate is 11.25% per year, compounded continuously and no other deposits or withdrawals are made.

The following pages are copies of your homework in order

The last two pages: You may choose one of the mini-projects to complete.

-- Unit 4 --
Exponential Growth and Decay

Homework

Directions: Classify each function as an exponential growth or an exponential decay. Sketch the curve.

1. $f(x)=\frac{1}{7}\cdot 6^x$

2. $f(x)=\frac{3}{2}\cdot\left(\frac{1}{4}\right)^x$

3. $f(x)=3\cdot\left(\frac{5}{2}\right)^x$

Directions: **(a)** Identify the parent function and **(b)** describe the transformations.

4. $f(x)=-\left(\frac{4}{3}\right)^{2(x-3)}+1$

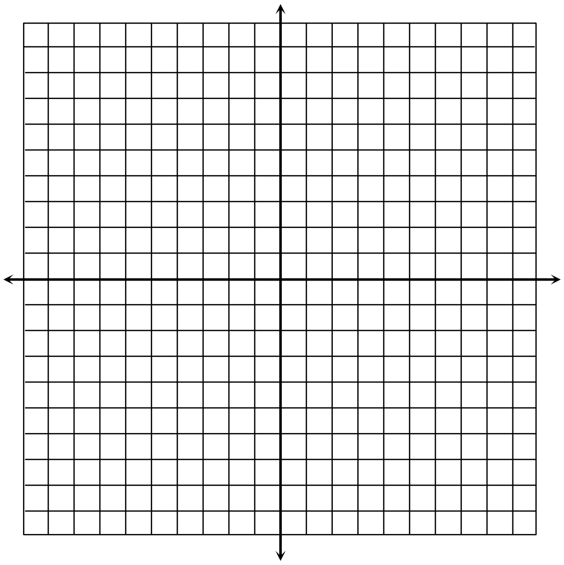
5. $f(x)=7^{-\frac{3}{2}x}+3$

6. $f(x)=\frac{1}{3}\cdot e^{-x}-9$

7. $f(x)=5\cdot\left(\frac{4}{5}\right)^{x+3}$

Directions: Graph each function, then identify its key characteristics.

8. $f(x)=\frac{1}{2}\cdot 5^{-x}-1$



Domain:

Range:

y-intercept:

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

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-- Unit 4 --
Growth and Decay

Homework

Exponential Growth and Decay

1. Aaron owns a rare baseball card. He bought the card for $7.50 in 1987 and its value increases by 6% each year. Write and use an exponential growth function to find the baseball card's value in 2015.

2. Jennifer started working at her job earning $6.25 per hour. Every six months, she gets a 3.25% raise. If Jennifer has worked at the job for 14 years, what is her hourly rate?

3. In 2005, the Summerville Journal had 110,000 subscriptions. The number of subscriptions subsequently decreased by 8% each year. Write and use an exponential decay function to find the number of subscriptions in 2022.

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8. An ice sculpture measures 52 inches and melts continuously by 3% per minute. Find the height of a sculpture after 15 minutes.

9. In 2002, a certain town recorded 15,300 acres of undeveloped land. Each year after, the amount of land decreased by 7% due to residential and commercial development. Find the approximate amount of undeveloped land in 2013.

Logistic Growth

10. The population of fish in a pond from 2001 to 2014 is modeled by the function below, where t is the years since 2001. Using the function, find the number of fish in the pond in 2014.

$$P(t) = \frac{1125}{1+12e^{-0.17t}}$$

11. The bears in Alaska are limited to a certain area to live due to the resources available for food and shelter. After t years, the number of bears living in the area is modeled by the function below. Using the function, find the number of bears after 17 years.

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HW

Compound Interest

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a) quarterly

b) weekly

2. Erica was given \$300 for her birthday and decided to put it in a savings account that earns 3.75% interest. If she makes no other deposits or withdrawals, find her account balance after ten years if the interest is compounded with the following frequencies.

a) semiannually

b) daily

3. A \$2,750 deposit was made to an account earning $2\frac{3}{4}\%$ annual interest compounded weekly. If no other deposits or withdrawals are made, find the balance of the account after nine years.

Continuous Compound Interest

7. Moises was given a \$1,500 signing bonus at his new job. He is going to invest this money in an account that earns 6% interest, compounded continuously. Find the account balance after ten years.

8. Suppose \$2,800 is deposited into an account at a 2.5% interest rate, compounded continuously. If there are no other deposits or withdrawals, find the account balance after 25 years.

9. Find the balance of an account after seven years if \$600 is deposited and the interest rate is 11.25% per year, compounded continuously and no other deposits or withdrawals are made.

STEM-ersion

w/ Exponential Functions

Name ______________________

Date ______________________

Financial Advisor

Jorge is working with a client who has received an inheritance of $50,000. The client wants to purchase mutual funds and wants to diversify the investments between four categories. They have asked him to invest at least $10,000 into each of the categories, but they'd like his recommendation for how to use the rest. With the formula:

$FV = PV(1+r)^t$ *where FV is future value, PV is present value, r is the rate and t is the time in years*

Jorge makes 30 year portfolio value estimates for each client with formulas for Best Case and Worst Case scenarios. Help Jorge decide where the other $10,000 should go and determine the projections for each case.

	Bond (Low Risk)	International (High Risk)	Small Cap (Medium Risk)	S & P 500 (Medium Risk)
Best Case	$PV(1+0.052)^t$	$PV(1+0.121)^t$	$PV(1+0.083)^t$	$PV(1+0.091)^t$
Worst Case	$PV(1+0.049)^t$	$PV(1-0.009)^t$	$PV(1+0.028)^t$	$PV(1+0.026)^t$

Evidence

Use this space to make any calculations and show work.

Analysis

of the evidence

Interpret the Evidence. What does it mean?

Conclusion

or Recommendation

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Exponential Function Modeling

Name ______________________________ Date ______________________________

KICKOFF

BACKGROUND

Raul is an executive for Major League Soccer. The league desires to expand in new markets, and he is evaluating the growth of the different cities in regards to population (potential total fans) and the GDP per capita (potential corporate sponsors and season ticket holders). He has exponential models for these statistics, and he would like to evaluate them in the next five years.

Which city appears to have the most promise for potential expansion?

	Cincinnati	Indianapolis	Phoenix	San Diego	Tampa
Population $p(x)$	$298{,}800e^{0.017t}$	$864{,}800e^{0.015t}$	$1{,}615{,}000e^{0.021t}$	$1{,}407{,}000e^{0.004t}$	$378{,}200e^{0.032t}$
Economy $f(x)$	$52{,}063e^{0.02t}$	$53{,}441e^{0.019t}$	$44{,}803e^{0.021t}$	$57{,}955e^{0.016t}$	$40{,}153e^{0.016t}$

HALF TIME

RESEARCH

Use this space to make any calculations and show work.

STOPPAGE TIME

ARGUMENTS

What are the two or three best choices to make and why?

DECISION

Make a decision and provide reasons to support it.

SHOOTOUT

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